Reg. No .... Name **26U604**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT09 : REAL ANALYSIS - 2**

(For Regular - 2023 Admission and Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Is every Riemann integrable function continuous? Justify your answer.
2. Discuss the differentiability of the function $f(x) = \begin{cases} 2, & x \le 1 \\ x, & x > 1 \end{cases}$ at $x = 1$
3. When is a partition P^* of $[a, b]$ said to be finer than another partition P of $[a, b]$?
4. Show that the series $\sum \frac{(-1)^n}{n} |x|^n$, converges uniformly in $[-1, 1]$.
5. Prove that $f(x) = \begin{cases} x^2 \, sin(1/x) & when \, x \neq 0 \\ 0 & when \, x = 0 \end{cases}$ is differentiable at $x = 0$.
6. Give an example of a function which is not Riemann integrable on $\mathbb{R}$.
7. Define the improper integral $\int_a^b f \, dx$, where a and b are the only points of infinite discontinuity.
8. Show that the series $\sum r^n \cos n^2\theta, \ 0 < r < 1$, converges uniformly for all real values of θ.
9. Determine the nature of discontinuity at the origin of the function
$$f(x) = \begin{cases} \dfrac{\sin 2x}{x} & when \, x \neq 0 \\ 0 & when \, x = 0 \end{cases}.$$
10. State and prove the symmetrical property of the Beta function.
11. Compute $\Gamma(-\frac{5}{2})$.
12. Show that the series $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$, converges uniformly for all real x.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Show that a constant function $f(x) = k$ is integrable on $[a, b]$ and $\int_a^b k \, dx = k(b - a)$.
14. If f and g are continuous at a point c, show that fg is also continuous at c.
15. Let f be differentiable at $c \in (a, b)$. If $f'(c) < 0$, show that f is decreasing at c.

16. Show that every continuous function is integrable.

17. Find the pointwise limit $f(x)$ of $\{f_n(x)\}$ where $f_n(x) = nx(1-x^2)^n,\ 0 \leq x \leq 1$.
Show that $\int_0^1 f_n(x)dx$ does not converge to $\int_0^1 f(x)\,dx$.

18. If n is a positive integer, prove that $2^n\Gamma(n+\frac{1}{2}) = 1.3.5...(2n-1)\sqrt{\pi}$.

19. Evaluate $\int_0^2 \frac{x^2\,dx}{\sqrt{2-x}}$.

20. Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{nx}{1+n^2x^2}$ is not uniformly convergent on any interval containing zero.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Show that the sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx,\ x \geq 0$ is uniformly convergent on any interval $[a, b]$, $a > 0$ but is only pointwise convergent on $[0, b]$.

22. Prove that a necessary and sufficient condition for the integrability of a bounded function f is that to every $\epsilon > 0$, there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta$, $U(P, f) - L(P, f) < \epsilon$.

23. Evaluate $\int_0^\infty x^3 e^{-x^2}\,dx$.

24. Show that a function which is continuous on a closed interval is also uniformly continuous on that interval.

(10 x 3 = 30)