Reg. No .... Name **26P463**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 4 : MATHEMATICS

COURSE : **24P4MATT20EL : THEORY OF WAVELETS**

(For Regular - 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. If N is divisible by 2^l, define $D^l : l^2(Z_N) \longrightarrow l^2(Z^l_{N/2})$. (U)
2. When we say there is a perfect reconstruction in the filter bank? (U)
3. If z=$(z(n))_{n\in Z}$ is square summable and $\alpha \in C$, prove that αz ,is square summable. (A)
4. Show that if $\sum_{n\in Z} w(n)$ converges absolutely, then $\lim_{n\to\infty} w(n) = 0$ and $\lim_{n\to\infty} w(-n) = 0$. (An)
5. If $z_1 w \in l^2(z)$, prove that $\left[u^l(z) * w\right] \cong u^l(\tilde{z}) * \tilde{w}$. (An)
6. Define the Inverse Fourier transform on $L2([-\pi,\pi))$. (An)
7. For any $w \in l^2(Z_N)$, define $\tilde{w}$. If $w = (1, 2, 0, -1) \in l^2(Z_4)$, find $\tilde{w}$. (U)
8. If N is divisible by 2^p, with the usual notations define f_l, g_l for $l = 1, 2, 3, \dots, p$. (An)
9. If $z = (z(n))_{n\in Z} \in l^2(Z)$, prove that $DoU(z) = z$. (An)
10. Prove that $\hat{z}(m) = \sqrt{N} < z, E_m >; 0 \le m \le N-1$. (An)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. (a) Define the Dirac delta function δ.
(b) For any $w \in l^2(Z_N)$ prove that $w * \delta = w$. (R)
12. i) Define delta function $'\delta'$.
(ii)Suppose $b \in l^1(Z)$ and define $T_b(z) = b * z$ for all $z \in l^2(Z)$. Then prove that $T_b : l^2(Z) \to l^2(Z)$ is a translation invariant linear transformation. (An)
13. Suppose H is a Hilbert space and $\{a_j\}_{j\in Z}$ is an orthonormal set in H .Then prove that $\{a_j\}_{j\in Z}$ is a complete orthonormal set if and only if f=$\sum_{j\in Z} < f, a_j > a_j$; for all f$\in H$. (U)
14. Suppose $u_1 v \in l^1(z)$ are such that $[R_{2k}v]_{k\in z} \cup [R_{2k}u]_{k\in z}$ is a first stage wavelet system for $l^2(z)$. Suppose also that $u(n) = v(n) = 0 \forall n < 0$ and $n > N-1$. Define $u_{(N)}, v_{(N)} \in l^2(Z_N)$ by $u_{(N)}(n) = u(n)$ and $v_{(N)}(n) = v(n)$ for $n = 0, 1, \cdots N-1$. Then prove that $[R_{2k}v_{(N)}]^{M-1}_{K=0} \cup [R_{2k}u_{(N)}]^{M-1}_{K=0}$ is a first stage wavelet basis for $l^2(z_n)$. (R)
15. Suppose $z, w \in l^2(Z_N)$. Prove that
(i) $(z * w) = \tilde{z} * \tilde{w}$.
(ii) $[D(z)]^{\sim} = D(\tilde{z})$, if N is even and
(iii) $[U(z)]^{\sim} = U(\tilde{z})$ (An)

16. Describe the first stage real shannon basis for $l^2(Z_N)$ if N is divisible by 4. (U)

17. Let $\hat{u} = (\sqrt{2}, 1, 0, 1)$ and $\check{v} = (0, 1, \sqrt{2}, -1)$
(a) Find u and v (A)
(b) Construct an orthonormal basis for $l^2(Z_4)$ using u and v

18. Suppose N is even and N=2M. Let $z \in l^2(Z_N)$ and $x, y, w \in l^2(Z_{N/2})$. Then prove that $D(z) * w = D(z * U(w))$ and $U(x) * U(y) = U(x * y)$. (An)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Describe real shannon wavelet system. (An)

20. Describe Daubechies's D_6 wavelet system for $l^2(Z)$. (An)

21. Let $\{a_j\}_{j \in Z}$ be an orthonormal set in a Hilbert space H. Then Prove that the following statements are equivalent (A)
i) $\{a_j\}_{j \in Z}$ is Complete
ii) For any f,g $\in H, < f, g >= \sum_{j \in Z} < f, a_j > \overline{< g, a_j >}$
iii) For any f $\in H, ||f||^2 = \sum_{j \in Z} | < f, a_j > |^2$.

22. (a) Suppose $z, w \in l^2(Z_N)$. Then prove that for each m,
$(z * w)^{\wedge}(m) = \hat{z}(m)\,\hat{w}(m)$
(b) Let z=(1,0,1,0) and w=(0,1,0,1) be two elements of $l^2(Z_4)$. Find $z * w$. (A)
(c) Find $(z * w)^{\wedge}$
(d) Find $\hat{z}$ and $\hat{w}$ and verify $(z * w)^{\wedge}(m) = \hat{z}(m)\hat{w}(m)$ for m=0,1,2,3.

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;