

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2026**SEMESTER 2 : MATHEMATICS****COURSE : 24P2MATT10 : MEASURE THEORY AND INTEGRATION***(For Regular 2025 Admission and Improvement/Supplementary 2024 Admission)*

Time : Three Hours

Max. Weights: 30

PART A**Answer any 8 questions****Weight: 1**

1. By using properties of outer measure , prove that the interval $[0, 1]$ is not countable. (A)
2. Define Lebesgue integral of a bounded measurable function over a set of finite measure. (U)
3. Define the integral of a non negative simple function over general measure space. (U)
4. State Monotone convergence theorem over general measure space. (R)
5. Prove that a real valued function that is continuous on its measurable domain is measurable. (A)
6. Let A be the set of all irrational numbers in $[0, 1]$. Prove that $m^*(A) = 1$. (A)
7. Show that the sum and product of two simple functions are again simple. (A)
8. Define what it means for a set to be measurable. (R)
9. Let (X, M, μ) be a measure space. Suppose $A, B \in M$ and $A \subset B$. Then prove that $\mu A \leq \mu B$. (A)
10. Define measurable functions over a general measure space. (U)

(1 x 8 = 8)**PART B****Answer any 6 questions****Weights: 2**

11. For a finite family $\{f_k\}_{k=1}^n$ of measurable functions with common domain E , show that the functions $\max\{f_1, f_2, \dots, f_n\}$ and $\min\{f_1, f_2, \dots, f_n\}$ are also measurable. (A)
12. Let f be a nonnegative measurable function on E . Then prove that $\int_E f = 0$ iff $f = 0$ a.e. on E . (A)
13. State and prove Beppo Levi's lemma over general measure space. (A)
14. Show that outer measure of an interval is its length. (A)
15. Let R be the collection of measurable rectangles in $X \times Y$ and for a measurable rectangle $A \times B$, define $\lambda(A \times B) = \mu(A) \cdot \nu(B)$. Then prove that R is a semiring and $\lambda : R \rightarrow [0, \infty]$ is a premeasure. (An)
16. State and prove Hahn's lemma. (A)
17. Show that outer measure is countable subadditive. (A)
18. Let $A \in X$ and $\{E_k\}_{k=1}^n$ be a finite disjoint collection of measurable sets. Then prove that $\mu^*(A \cap [\cup_{k=1}^n E_k]) = \sum_{k=1}^n \mu^*(A \cap E_k)$. (A)

(2 x 6 = 12)

PART C
Answer any 2 questions

Weights: 5

19. Construct Cantor set and show that it is closed, uncountable set of measure zero. (An)
20. Prove that the class M of μ^* measurable sets is a σ algebra. If $\bar{\mu}$ is the restriction of μ^* to M , then prove that $\bar{\mu}$ is a complete measure on M . (A)
21. A) Let f and g be integrable over E . Then show that for any α and β , $\alpha f + \beta g$ is integrable over E and
 (i) $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.
 (ii) If $f \leq g$ on E , then $\int_E f \leq \int_E g$. (An)
- B) Let f be integrable over E . Suppose A and B are disjoint measurable subsets of E . Then prove that $\int_{A \cup B} f = \int_A f + \int_B f$.
22. State and prove Vitali Convergence theorem. (An)
- (5 x 2 = 10)**

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
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Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;