Name **26P237**

Reg. No ....

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2026

SEMESTER 2 : MATHEMATICS

COURSE : **24P2MATT08 : ALGEBRA -II**

(For Regular 2025 Admission and Improvement/Supplementary 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. True or False: $\mathbb{R}$ is a splitting field over $\mathbb{Q}$.Justify. (An, CO 4)
2. Show that $\mathbb{Q}\left(\sqrt{2}\right) = \mathbb{Q}(3+\sqrt{2})$ (An, CO 2)
3. True or False: $GF(19)$ is perfect.Justify your answer. (A, CO 4)
4. Is i a primitive 4th root of unity? Justify. (A, CO 3)
5. Are i and $-i$ conjugate over $\mathbb{R}$? Over $\mathbb{C}$? Justify your answer. (U, CO 3)
6. Find all zeroes of the polynomial $f(x) = x^2 + 1$ in $\mathbb{Z}_2$. (A, CO 1)
7. Find all $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x]/ < x^3 + x^2 + c >$ is a field? (An, CO 1)
8. Consider the evaluation homomorphism $\phi_5 : \mathbb{Q}[x] \to \mathbb{R}$. Find six elements in the kernel of the homomophism ϕ_5. (A, CO 1)
9. Find all irreducible polynomials of degree 2 in $\mathbb{Z}_2[x]$. (E, CO 2)
10. Show that $R[x]/\langle x^2+1\rangle \cong \mathbb{C}$. (An, CO 2)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that the elements of $GF(p^n)$ are precisely the zeroes in $\overline{\mathbb{Z}_p}$ of the polynomial $x^{p^n} - x$ in $\mathbb{Z}_p[x]$ (An, CO 3)
12. Show that if F is a field of prime characteristic p with algebraic closure $\overline{F}$, then $x^{p^n} - x$ has p^n distinct zeroes in $\overline{F}$. (An, CO 3)
13. Show that $\mathbb{Q}(\sqrt{3}+\sqrt{7}) = \mathbb{Q}(\sqrt{3}, \sqrt{7})$. (E, CO 2)
14. Show that a field F is algebraically closed if and only if every nonconstant polynomial in $F[x]$ factors in $F[x]$ into linear factors. (A, CO 2)
15. Use Fermat's theorem to evaluate $\phi_3(x^{231} + 3x^{117} - 2x^{53} + 1)$ in $\mathbb{Z}_5$. (An, CO 1)
16. Determine $G(\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q})$? (E, CO 4)
17. Show that $x^4 - 22x + 1$ is irreducible over $\mathbb{Q}$. (E, CO 1)
18. State the Isomorphism Extension Theorem.Using this show that any two algebraic closures of a field F are isomorphic under an isomorphism leaving each element of F fixed. (An, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. (a). If E is a finite extension of F, then show that $\{E : F\}$ divides $[E : F]$.
(b). Show that $\alpha \in \overline{F}$ is separable over F if and only if $irr(\alpha, F)$ has all zeroes of multiplicity 1. (E, CO 4)

20. State and prove the Conjugation Isomorphisms Theorem for field theory. (An, CO 3)

21. Stating the necessary lemmas, establish the existence and uniqueness of $\mathbf{GF}(p^n)$, the Galois field of order p^n. (E, CO 3)

22. (a). State and prove the division algorithm for $F[x]$.
(b). State and prove the factor theorem. (E, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain ring of polynomials, polynomial factorisation and the ideal structure in F{x}	E	6, 7, 8, 15, 17, 22	12
CO 2	Comprehend the concept of field extension and the types of extensions.	E	2, 9, 10, 13, 14	7
CO 3	Analyze finite fields and field automorphisms.	E	4, 5, 11, 12, 20, 21	16
CO 4	Analyze splitting fields, separable extensions and the main theorem of Galois theory	E	1, 3, 16, 18, 19	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;