Reg. No .... Name **26U235**

END SEMESTER EXAMINATION - APRIL 2026

SEMESTER 2 - INTEGRATED M.Sc. PROGRAMME COMPUTER SCIENCE- DATA SCIENCE

COURSE : **21UP2CPCMT02 - MATHEMATICS - II - LINEAR ALGEBRA**

(For Regular - 2025 Admission and Improvement/Supplementary 2024/2023/2022/2021 Admissions)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8

1. Give matrix representarion for the following operator ; $T \in L(F^3)$ defined by T(x,y,z) = (2x+y, 5y+3z, 8z)
2. Define linear independence and linear dependence.
3. Suppose T is linear map from V To W .Then prove that T(0) =0.
4. Define linear functional on a vector space.
5. Evaluate (2+3i)(4+5i).
6. Give the matrix representation of the vector $2\text{-}7x+5x^2$ with respect to the standard basis of $P_3(R)$.
7. Define innerproduct space.
8. Prove the following results;

   1. < v,0 > = 0 for every u ∈ V
   2. < u,v+w > = < u,v > + < u,w > for all u,v,w ∈ V

9. If $x,y \in F^n$,then prove that x+y = y+x.
10. Explain diagonal of a matrix and upper triangular matrix with suitable example.

(1 x 8 = 8 Weight)

PART B

Answer any 6

11. Prove that every orthonormal list of vectors are linearly independent
12. Check whether $T \in L(P^1)$ defined by T(at+b) = (a+2b)t +(4a+3b) is diagonalizable either over the standard basis of P^1 or with respect to the basis -t+1,5t+10 of P^1.Give valuable reason for your answers.
13. Give matrix representarion for the following operators

    1. $T \in L(F^3)$ defined by T(x,y,z) = (2x+y, 5y+3z, 8z)
    2. $T \in L(F^2)$ defined by T(x,y) = (2x+3y, 5x)

14. State and Prove Parallelogram inequality
15. Prove that suppose U_1,..........,U_m are subspaces of V. Then U_1 +..........+U_m is a direct sum if and only if the only way to write 0 as a sum u_1+........+u_m , where each u_j is in U_j , is by taking each u_j equal to 0.
16. Check whether the list of vectors (1,1),(1,-1) is basis in F^2
17. Suppose $D \in L(P(R),P(R))$ is the differentiation map defined by D_p = p' and $T \in L(P(R),P(R))$ is the multiplication by x^2 defined by $T_p(x) = x^2p(x)$.Then prove that multiplication on linear map is not commutative.

18. Find a basis of $P_2(R) \times R^2$

(2 x 6 = 12 Weight)

PART C

Answer any 2

19. Prove that Every spanning list in a vector space can be reduced to a basis of the vector space and also prove that every linearly independent list of vectors in a finite dimensional vector space can be extended to a basis of the vector space.

20.
    1. Check whether $T \in L(F^2)$ defined by T(x,y) = (41x+7y, -20x+74y) are diagonalizable either over the standard basis of F^2 or with respect to the basis (1,4),(7,5) of F^2
    2. Check whether $T \in L(P^1)$ defined by T(at+b) = (a+2b)t +(4a+3b) is diagonalizable either over the standard basis of P^1 or with respect to the basis -t+1,5t+10 of P^1. Give valuable reason for your answers.

21. Prove the following;

    1. Let $T \in L(V,W)$ .Then T is injective if and only if NullT ={0}
    2. If $T \in L(V,W)$ then range T is a subspace of W.

22. Find an Orthonormal basis of $P_2(R)$,where the innerproduct is given by < p,q > = $\int_{-1}^{1} p\left(x\right)q\left(x\right)\, dx$

(5 x 2 = 10 Weight)