Reg. No .... Name **26P220**

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2026

SEMESTER 2 : MATHEMATICS

COURSE : **24P2MATT07 : COMPLEX ANALYSIS**

(For Regular 2025 Admission and Improvement/Supplementary 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. State Hadamard's three circles theorem. (R, CO 4)
2. State and prove Cauchy's estimate. (U)
3. Evaluate 1) $(0,1,i,-1)$ 2) $(i-1,\infty,1+i,0)$ (An)
4. Define homotopy and fourth version of Cauchy's theorem. (R)
5. State two necessary and sufficient conditions for a function to be convex. (R)
6. Show that a Mobius transformation takes circles onto circles. (R)
7. For the $f(z) = \frac{log(z+1)}{z^2}$ has an isolated singularity at $z=0$. Determine its nature; if it is a removable singularity define $f(0)$ so that f is analytic at $z=0$; if it is a pole find the singular part (An)
8. Evaluate $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$ where n is a positive integer and $\gamma(t) = e^{it}, 0 \le t \le 2\pi$ (A)
9. Prove that $|e^z| = exp(Rez)$ (A)
10. For the $f(z) = \frac{cosz-1}{z}$ has an isolated singularity at $z=0$. Determine its nature; if it is a removable singularity define $f(0)$ so that f is analytic at $z=0$; if it is a pole find the singular part. (An, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If f has an isolated singularity at a then prove that the point $z=a$ is a removable singularity if and only if $\lim_{z\to a} f(z) = 0$ (An, CO 3)
12. State and prove Morera's theorem. (R)
13. State and prove second version of maximum modulus theorem. (U)
14. If $\gamma : [0,1] \to \mathbb{C}$ is a closed rectifiable curve and $a \notin \gamma$ then $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer. (A)
15. Define symmetric points. Explain symmetric points w.r.t a straight line (A)
16. Show that $f(z) = |z|^2$ has a derivative only at the origin (E)
17. Prove that a function $f : [a,b] \longrightarrow R$ is convex if and only if the set $A = \{(x,y)/a \le x \le b$ and $f(x) \le y\}$ is convex (An, CO 4)
18. Obtain the Laurent's series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region a) $|z| \le 2$ b) $2 \le |z| \le 3$ c) $|z| \ge 3$ (A, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. If $T(z) = \frac{az+b}{cz+d}$ show that $T(\mathbb{R}_\infty) = \mathbb{R}_\infty$ if and only if we can choose a, b, c, d to be real numbers. (An)

20. Show that a differentiable function f on $[a, b]$ is convex if and only if f' is increasing. (R, CO 4)

21. Show that for $a \geq 1, \int_0^\infty \frac{d\theta}{a+cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$ (A, CO 3)

22. Let γ be a rectifiable curve and suppose ϕ is a function defined and continuous on γ. For each $m \geq 1$ let $F_m(z) = \int_\gamma \phi(w)(w-z)^{-m} dw$ for $z \notin \gamma$. Then show that for each F_m is analytic on $\mathbb{C} - \gamma$ and $F_m'(z) = mF_{m+1}(z)$ (U)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 3	Represent analytic functions as power series	U	10, 11, 18, 21	10
CO 4	Identify zeros and classify singularities of complex function	U	1, 17, 20	8

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;