

**B A, B SC, B COM DEGREE END SEMESTER EXAMINATION - MARCH 2026****UGP (HONS.) SEMESTER 4 : DISCIPLINE SPECIFIC COURSE****COURSE: 24UMATDSC204 - DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS***(For Regular 2024 Admission)*

Time: 2 Hours

Max. Marks: 70

**PART A****(Each question carries 2 marks. Attempt any five. Maximum 10 marks)**

1. Solve:  $\frac{dy}{dx} = \frac{x^2}{y^2}$ . [CO1]
2. Check whether  $(y^3 - 2xy)dx + (x^2 - 3xy^2)dy = 0$  is exact. [CO1]
3. Solve the linear DE:  $\frac{dy}{dx} + y \cot x = \cos^2 x$ . [CO1]
4. Solve Clairaut's equation  $y = xp + \ln(1 + p)$ . [CO2]
5. Find the gradient of  $f(x, y, z) = e^{xyz}$  at the point  $(1,0,2)$ . [CO3]
6. Find the directional derivative of  $f(x, y, z) = x^2 + yz$  at  $(1,1,1)$  in the direction of vector  $(2, -1, 2)$ . [CO3]
7. Compute  $\text{curl } F$ , where  $F = (yz)i + (xz)j + (xy)k$ . [CO3]
8. Evaluate  $\int_C (x^2 + y)dx$ , where  $C$  is the line segment joining  $(1,1)$  and  $(3,5)$ , oriented from  $(1,1)$  to  $(3,5)$ . [CO4]

**PART B****(Each question carries 5 marks. Attempt any six. Maximum 30 marks)**

9. Solve:  $(y \cos x - x \sin y)dx + (x \cos y - y \sin x)dy = 0$ . [CO1]
10. Solve the DE:  $(x^2 + y^2)p = xy$ , where  $p = \frac{dy}{dx}$ . [CO2]
11. Solve Lagrange's equation:  $y = xp + e^p$ , where  $p = \frac{dy}{dx}$ . [CO2]
12. If  $F = (x^2z)i + (y^2x)j + (z^2y)k$ , evaluate  $\text{div } F$  and  $\text{curl } F$ . [CO3]
13. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 - z = 4$  at the point  $(2,1,2)$ . [CO3]

14. Evaluate  $\int_C (x^2 dy - y^2 dx)$  where  $C$  is the circle  $x^2 + y^2 = 1$  traversed counter-clockwise. [CO4]
15. Find the divergence and curl of  $4x^3y^2\hat{i} - 6xyz^3\hat{j} + 9y^2z\hat{k}$ . [CO4]
16. Evaluate  $\int_C 2xydx + (x^2 + y^2)dy$  along the circular arc  $C$  given by  $x = \cos t, y = \sin t; 0 \leq t \leq \frac{\pi}{2}$ . [CO5]

### PART C

(Each question carries 15 marks. Attempt any two. Maximum 30 marks)

17. a) Solve the initial value problem  $\frac{dy}{dx} + \frac{y}{x} = x^3, y(1) = 2$ . (8 marks)
- b) Solve  $(y - x)dx + (x + y)dy = 0$ . (7 marks) [CO1, CO2]
18. a) Solve the Clairaut's differential equation  $y = xp + p^3$ . (8 marks)
- b) Solve the PDE  $p^2 - q^2 = 1$ , where  $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ . (7 marks) [CO2]
19. a) Find the maximum directional derivative of  $f(x, y, z) = x^2y^2 + y^2z^2 + z^2x^2$  at  $(1,1,1)$  and the direction in which it occurs. (8 marks)
- b) If  $F = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ , show that  $F$  is conservative and find its scalar potential. (7 marks) [CO3]
20. a) A particle moves under the force field  $F(x, y) = (x^2y + y)\hat{i} + (x^2 + 2y)\hat{j}$ . Evaluate the work done in moving the particle along the boundary of the triangle with vertices  $(0,0), (1,0)$ , and  $(1,2)$ , taken counter-clockwise. (8 marks)
- b) Find the gradient of the function  $f(x, y, z) = x^2y + y^2z + z^2x$  at the point  $(1,2, -1)$ . (7 marks) [CO4, CO5]