Reg. No ....   Name **26U623**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT10 : COMPLEX ANALYSIS**

(For Regular 2023 Admission and Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours   Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that an antiderivative of a given function $f(z)$ is unique except for an additive constant.
2. Give an example of a simple pole.
3. Obtain the Taylor series of $\cos z$ about $z = -\pi$.
4. Find the real and imaginary parts of the function $f(z) = z^2$.
5. Express $\frac{z^2-2z+3}{z-2}$ in to the powers $(z-2)$.
6. Show that limit of $f(z)$ at $z = z_0$ is unique, if it exists.
7. Obtain the Taylor series of e^z about $z = 1$.
8. Write the Laurent series expansion of $f(z) = \dfrac{\sin z}{\pi - z}$ about $z = \pi$.
9. Use definition to evaluate $f'(0)$, where $f(z) = \bar{z}$.
10. Write all the singularities of $\frac{1}{\sin(\pi/z)}$.
11. State Cauchy-Goursat theorem.
12. State Jordan curve theorem.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Prove that $\sum\limits_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.
14. State Laurent's theorem and compute the Laurent series expansion of $f(z) = \frac{1+2z^2}{z^3+z^5}$ about $z = 0$.
15. Compute the residue of $f(z) = \dfrac{1}{z^2(1+z)}$ at $z = 0$.
16. Verify Cauchy-Riemann equations are satisfied everywhere by the function $f(z) = z^2$.
17. Let C denote a contour of length L, and suppose that a function $f(z)$ is piecewise continuous on C. If M is a nonnegative constant such that $|f(z)| \le M$ for all points z on C at which $f(z)$ is defined , then prove that $\left| \int\limits_C f(z)dz \right| \le ML$
18. Use Cauchy integral formula to evaluate $\displaystyle\int\limits_{|z|=1} \frac{e^{2z}}{z^4} dz$.

19. Find the residues of the singularities of $f(z) = \dfrac{z^3 + 2z}{(z-1)^3}$.

20. Find the harmonic conjugate of $u(x, y) = y^3 - 3x^2y$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Derive Cauchy-Riemann equations.

22. State and prove Morera's theorem.

23. Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \dfrac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and nonzero at z_0 and $\underset{z=z_0}{Res} f(z) = \frac{\phi^{(m-1)}(z_0)}{(m-1)!}$

24. Suppose that $z_n = x_n + iy_n$ and $z = x + iy$. Show that $\lim_{n\to\infty} z_n = z$ if and only if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.

(10 x 3 = 30)