Reg. No........................ Name......................... 26U4141

BA, B.SC, B.COM DEGREE END SEMESTER EXAMINATION - MARCH 2026

UGP (HONS) SEMESTER - 4 : DISCIPLINE SPECIFIC COURSE

COURSE : 24UMATDSC203 : FOUNDATIONS OF ANALYSIS AND ALGEBRA

(For Regular 2024 Admission)

Time : 2 Hours Max. Marks : 70

Section A

Each Question Carries 2 Marks each.
A Maximum of 10 Marks can be scored in this section.

(1) Define a countable set. Give an example. [**CO1**]

(2) Define the absolute value of a real number a. [**CO2**]

(3) Write the polar forms of the fourth roots of unity. [**CO3**]

(4) Show that for any complex number z,

(a) $Re(iz) = -Im\, z$
(b) $Im(iz) = Re\, z$ [**CO3**]

(5) Is $< \mathbb{R}^*, . >$ a subgroup of $< \mathbb{R}, + >$? Justify your answer. [**CO4**]

(6) Give an example of a non abelian group. [**CO4**]

(7) Define ring with unity. Give an example of a ring without unity. [**CO5**]

(8) Let R_1 and R_2 be rings. Show by means of an example that a homomorphism between the groups $< R_1, + >$ and $< R_2, + >$ need not be a homomorphism between the rings R_1 and R_2. [**CO5**]

Section B

Each Question Carries 5 Marks each.
A Maximum of 30 Marks can be scored in this section.

(9) Suppose that S and T are sets such that $T \subset S$. Prove that if S is finite, then T is also finite. [**CO1**]

(10) Show that $\mathbb{N} \times \mathbb{N}$ is denumerable. [**CO1**]

(11) State and prove the triangular inequality for any two complex numbers z_1 and z_2 [**CO3**]

(12) Find the polar form of the number $-1 - i$. [**CO3**]

(13) Let G be a group and let $a \in G$. Show that $H = \{a^n | n \in \mathbb{Z}\}$ is a subgroup of G. What is this subgroup called and how is it usually denoted? [**CO4**]

(14) Show that the set $M_{m\times n}(\mathbb{R})$ of all $m \times n$ matrices under matrix addition is a group. Is it an abelian group? Justify your answer. **[CO4]**

(15) Prove that $< \mathbb{Q}, +, . >$ is a field. **[CO5]**

(16) Let R be a ring with unity. If $n.1 \neq 0$ for all $n \in \mathbb{Z}^+$, show that R has characteristic 0. Show that if $n.1 = 0$ for some $n \in \mathbb{Z}^+$, then the smallest such n is the characteristic of R. **[CO5]**

Section C
Each Question Carries 15 Marks each.
A Maximum of 30 Marks can be scored in this section.

(17) (a) Show that there does not exist a rational number r such that $r^2 = 2$.

(b) Show that the union of a countable collection of countable sets is countable. **[CO1]**

(18) (a) Find the third roots of $-8i$. Also identify the principal third root of the same. **[CO3]**

(b) If $z = re^{i\theta}$, prove that $z^n = r^n e^{in\theta} \quad (n = 0, \pm 1, \pm 2, \ldots)$

[CO3]

(19) (a) Draw the group tables of the group Z_4 and the Klein-4 group V. Are both groups cyclic? Justify your answer. Also draw the subgroup lattice diagrams of both these groups.

(b) State and prove a necessary and sufficient condition for a subset H of a group G to be a subgroup of G. **[CO4]**

(20) (a) If p is a prime, prove that $\mathbb{Z}_p$ is an integral domain. Is the result true if p is not prime? Justify your answer.

(b) State and prove a necessary and sufficient condition for the cancellation laws, with respect to multiplication, to hold in a ring. **[CO5]**