

Reg. No

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026**SEMESTER 6 : MATHEMATICS****COURSE : 19U6CRMAT12 : FOURIER SERIES, LAPLACE TRANSFORMS AND METRIC SPACES***(For Regular - 2023 Admission and Supplementary 2022/2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. State the Dirchlet's conditions.
2. Any metric space has two open subsets. True or False. Justify.
3. Give an example of a sequence which is not convergent but has a limit point for its set of points.
4. Find the set of all limit points of the sets \mathbb{N} and \mathbb{Q} in $X = \mathbb{R}$ with the usual metric.
5. Find the inverse Laplace transform of $\frac{s+1}{s^2+2s}$.
6. Write the formula for Fourier series of a function with period $2l$ defined in the interval $[0, 2l]$
7. Define a Cauchy sequence. Give example.
8. Define Laplace transform and find the Laplace transform of $\cos ax$
9. Define convergence of a sequence in a metric space with respect to metric and open spheres.
10. Find the Laplace transform of $\sin 2t \cos 3t$.
11. Define periodic functions. Check whether the function $\sin ax$ is periodic.
12. Define distance of a point from a set and diameter of a set in terms of metric.

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Prove that each open sphere is an open set in a metric space.
14. Prove that if $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X , then there exists a point in X which is not in any of the A_n 's
15. Find the inverse Laplace transform of $\frac{21s-33}{(s+1)(s-2)^3}$
16. Let X and Y be metric spaces and f a mapping of X into Y . Prove that if f is continuous, then $f^{-1}(G)$ is open in X whenever G is open in Y .
17. Let A and B be two subsets of a metric space X . Then prove that $\text{Int}(A) \cup \text{Int}(B) \subseteq \text{Int}(A \cup B)$. Give an example to show that the reverse inclusion is not true.
18. Find the Fourier sine and cosine series of $f(x) = 1 + x, 0 \leq x \leq 1$.
19. Obtain Fourier series for the function $f(x) = \begin{cases} \pi x & 0 \leq x \leq 1 \\ \pi(2-x) & 1 \leq x \leq 2 \end{cases}$
20. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Let X be a metric space. Prove that
- 1) the empty set \emptyset and the full space X are closed.
 - 2) A subset F of X is closed \Leftrightarrow its complement F' is open.
22. a) Solve $(D^2 - D - 2)x = 20 \sin 2t$, $x(0) = -1$, $x'(0) = 2$
b) Apply convolution theorem to evaluate $L^{-1} \left\{ \frac{1}{s(s^2+4)} \right\}$
23. Find the half range Fourier sine and cosine series of the
function $f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$
24. State and prove the necessary and sufficient sequential criterion for a function $f : X \rightarrow Y$ to be continuous at a point $x_0 \in X$, where X and Y are metric spaces.

(10 x 3 = 30)