Reg. No .... Name **26P436**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 4 : MATHEMATICS

COURSE : **24P4MATT18EL : PROBABILITY THEORY**

(For Regular 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. A die is rolled twice. Let all the elementary events in $\Omega = (i, j) : i, j = 1, 2, \ldots, 6$ be assigned the same probability. Let A be the event that the first throw shows a number ≤ 2, and B be the event that the second throw shows atleast 5. Find $P(A \cup B)$. (A)
2. State Cauchy Schwartz inequality and Minkowski's inequality. (U)
3. A fair coin is tossed three times. Let A be the event that atleast one head shows up in three rows. Find $P(A)$. (A)
4. Define moments of a random variable. (R)
5. Does $F(x) = \frac{1}{\pi} tan^{-1}(x), -\infty < x < \infty$ define a DF? (A)
6. Let (X, Y) be a rv with joint pdf given by $f(x, y) = \begin{cases} e^{-(x+y)}, x > 0, y > 0 \\ 0, otherwise \end{cases}$. Find $F(x, y)$. (A)
7. State WLLN and SLLN. (U)
8. Let X be any rv. Show that $P(X = a) = \lim_{t \to a, t < a} P(t < X \leq a)$. (A)
9. Define almost sure convergence. (R)
10. Define mutually or completely independent rv's. (R)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. State Khinchin's theorem.Examine if WLLN holds for the $\{X_n\}$ of iid rv's with $P(X_i = (-1)^{k-1}k) = \frac{6}{\pi^2 k^2}, k = 1, 2, 3, \ldots, i = 1, 2, 3, \ldots$. (A)
12. Let $\{A_n\}$ be a non increasing sequence of events in S.Then prove that $lim_{n\to\infty} P(A_n) = P(lim_{n\to\infty} A_n) = P(\cap_{n=1}^{\infty} A_n)$. (A)
13. Suppose that $P(X \geq x)$ is given for a rv X of the continuous type for all x.How will you find the corresponding density function? In particular , find the density function if $P(X \geq x) = \begin{cases} 1, x < 0 \\ (1 + \frac{x}{\lambda})^{-\lambda}, x \geq 0 \end{cases}$ where $\lambda > 0$ is a constant. (A)
14. Let X be a rv with pdf $f(x) = \begin{cases} 1, 0 \leq x \leq 1 \\ 0, otherwise \end{cases}$. Let T ={$\frac{1}{3} < x \leq \frac{1}{2}$}. Find the truncated pdf wrt T, its mean and its variance. (A)
15. Suppose that the rv (X, Y) is uniformly distributed over the region R ={$(x, y) : 0 < x < y < 1$}. Find the $cov(X, Y)$. (A)
16. State and prove total probability rule. (A)
17. Let (X_n, X) has joint pmf $P(X_n = 0, X = 0) = 0, P(X_n = 1, X = 0) = \frac{1}{2}, P(X_n = 0, X = 1) = \frac{1}{2}, P(X_n = 1, X = 1) = 0.$ Then show that $X_n \overset{P}{\nrightarrow} X$ but $X_n \xrightarrow{L} X$. (An)
18. Define MGF. Let X have pdf $f(x) = \begin{cases} \frac{1}{2} e^{\frac{-x}{2}}, x > 0 \\ 0, otherwise \end{cases}$.The find the MGF of X. (An)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $h(X)$ be a non negative Borel measurable function of a rv X. If $E(h(X))$ exists, then show that for every $\epsilon > 0, P(h(X) \geq \epsilon) \leq \frac{E(h(X))}{\epsilon}$. (An)

20. a) Let $X_n \xrightarrow{r} X$ for some $r > 0$. Then show that $X_n \xrightarrow{P} X$.
b) Let $\{X_n\}$ be a sequence of random variables with pmf $P(X_n = 0) = 1 - \frac{1}{n^r}$ and $P(X_n = n) = \frac{1}{n^r}, r > 0, n = 1, 2, 3, \ldots$.Show that $X_n \overset{r}{\nrightarrow} 0$ but $X_n \xrightarrow{P} 0$. (A)

21. Show that the function defined by $f(x, y, z, u) = \frac{24}{(1+x+y+z+u)^5}, x > 0, y > 0, z > 0, u > 0$, and $= 0, otherwise$ is a joint pdf . Also find $P(X > Y > Z > U)$. (A)

22. a) State and prove Bayes theorem.
b) In answering a question on a multiple choice test, a candidate either knows the answer with probability $p, (0 \leq p < 1)$ or does not know the answer with probability $1 - p$. If he knows the answer, he puts down the correct answer with probability 0.99, whereas if he guesses, the probability of his putting down the correct result is $\frac{1}{k}$ (k choices to the answer). Find the conditional probability that the candidate knew the answer to a question , given that he has made the correct answer. Show that this probability tends to 1 as $k \to \infty$. (An)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;