Reg. No .... Name **26U642**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT11 - LINEAR ALGEBRA AND GRAPH THEORY**

(For Regular - 2023 Admission and Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define linearly dependent and linearly independent set of vectors in a vectorspace.
2. Determine whether the transformation **T: V → W** defined by T(v) = 0 for all vectors v in **V** is linear.
3. Define linear combination in a vectorspace.
4. Prove that if A and B are similar then A^T and B^T are similar.
5. Draw the graph corresponding to the adjacency matrix.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

6. Explain M augmenting path with an example
7. Explain the travelling salesman's problem
8. Prove that if A and B are similar then A^2 and B^2 are similar.
9. What is the maximum number of vertices on a graph that has 35 edges and every vertex has degree ≥ 3?
10. State and prove first theorem on graph theory.
11. Determine which of the following sets are spanning sets for $\mathbf{R^2}$, considered as column matrices:

(a) $\mathbb{S}_1 = \left\{ \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

(b) $\mathbb{S}_2 = \left\{ \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad f_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(c) $\mathbb{S}_3 = \left\{ \boldsymbol{f}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{f}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$

12. Explain the Chinese postman problem.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Let α be a scalar and **u** a vector in a vector space V.
If a α ⊙ **0** = **0**, then either α = 0 or **u = 0.**

14. Let v be a vertex of a connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w different from v, such that v is on every $u-w$ path

15. a) Draw K_5 and mark a maximum matching in the graph
b) Draw all trees with 6 vertices
c) Is K_n Eulerian

16. Determine whether the transformation **L** is linear if **L: P_3 → P_2** is defined by L($a_3t^3 + a_2t^2 + a_1t + a_0$) = $3a_3t^2 + 2a_2t + a_1$ where a_i (i = 0,1,2,3) denotes real number.

17. Find the matrix representation with respect to the standard basis in $\mathbf{R^2}$ and the standard basis C ={ t^2, t, 1 } in $\mathbf{P_2}$ for the linear transformation **T: $\mathbf{R^2}$ → $\mathbf{P_2}$** defined by

$$\boldsymbol{T}\begin{bmatrix} a \\ b \end{bmatrix} = 2at^2 + (a+b)t + 3b$$

18. a)Prove that it is impossible to have group of nine people such that each one knows exactly five others in the group.
b) Draw a graph which is not Euler but having an Euler trail.

19. Prove that Every basis for a finite-dimensional vector space must contain the same number of vectors.

20. Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. State and prove Dirac theorem.

22. Let G be a graph with n vertices.Then prove that the following statements are equivalent
(a) G is a tree
(b) G is acyclic graph with $n-1$ edges
(c) G is a connected graph with $n-1$ edges

23. Prove that for any linear transformation T from an n-dimensional vector space V to W, sum of rank of T and nullity of T is n, the dimension of the domain.

24. Use row rank to determine whether the following sets are linearly independent or not.
a) $\{[1 \quad 1 \quad 0], [1 \quad -1 \quad 0]\}$.
b) $\{[1 \quad 2 \quad 3], [-3 \quad -6 \quad -9]\}$.
c) $\{[10 \quad 20 \quad 20], [10 \quad -10 \quad 10], [10 \quad 20 \quad 10]\}$.

(10 x 3 = 30)