Reg. No .... Name **26P420**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2026

SEMESTER 4 : MATHEMATICS

COURSE : **24P4MATT17 : DIFFERENTIAL GEOMETRY**

(For Regular 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. What are principle curvature directions of s at p related to L_p. (U, CO 4)
2. Define graph. (U, CO 1)
3. Give Frenet formula for a plane curve. (U, CO 3)
4. Show that parallel vector fields form a vector space. (A, CO 2)
5. Draw the level curve of $f(x,y) = x^2 - y^2$ at $c = 1$ (A, CO 1)
6. Find the velocity, the acceleration, and the speed of parametrized curve $\alpha(t) = (\cos 3t, \sin 3t)$. (A, CO 2)
7. Find the integral curve through $p = (x_1, x_2) = (1, 1)$ of the vector field $\mathbb{X}(p) = (p, -p)$. (A)
8. Show that if $\alpha : I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$ for all $t \in I$. (A, CO 2)
9. Prove that $d\varphi(\mathbf{v})$ is independent of α. (A, CO 4)
10. Prove that $D_{\overline{u}}(\overline{X} + \overline{Y}) = D_{\overline{u}}\overline{X} + D_{\overline{u}}\overline{Y}$. (A, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find global parametrizations the plane curve $f^{-1}(c)$, oriented by $\nabla f / \|\nabla f\|$ where $f(x_1, x_2) = ax_1 + bx_2, \quad (a, b) \neq (0, 0)$. (A, CO 3)
12. Show that a parametrized curve α in the unit n-sphere $x_1^2 + \cdots + x_{n+1}^2 = 1$ is a geodesic if and only if it is of the form $\alpha(t) = e_1 \cos at + e_2 \sin at$ for some orthogonal pair of unit vectors $\{e_1, e_2\}$ in $\mathbb{R}^{n+1}$ and some $a \in \mathbb{R}$. (An, CO 2)
13. Let $S = f^{-1}(c)$ be an n-surface in $\mathbb{R}^{n+1}$, where $f : U \to \mathbb{R}$ is such that $\nabla f(q) \neq 0$ for all $q \in S$, and let $\mathbb{X}$ be a smooth vector field on U whose restriction to S is a tangent vector field on S. If $\alpha : I \to U$ is any integral curve of X such that $\alpha(t_0) \in S$ for some $t_0 \in I$, then prove that $\alpha(t) \in S$ for all $t \in I$. (A, CO 1)
14. Compute $\nabla_v f$ where $f(x_1, x_2) = x_1^2 - x_2^2$, $v = (1, 1, \cos\theta, \sin\theta)$. (A, CO 3)
15. Prove that the local parameterization is unique up to a reparameterization. (An, CO 2)
16. Let V be a finite dimensional vector space with dot product and let $L : V \to V$ be a self-adjoint linear transformation on V. Let $S = \{v \in V : v \cdot v = 1\}$ and define $f : S \to \mathbb{R}$ by $f(v) = L(v) \cdot v$. Suppose f is staionary at $v_0 \in S$. Prove that $L(v_0) = f(v_0)v_0$. (An, CO 4)
17. Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$. Prove that the Gauss-Kronecker curvature $K(p)$ of S at p is non-zero for all $\in S$ if and only if the second fundamental form $\mathscr{S}_p$ of S at p is definite for all $p \in S$. (An, CO 4)

18. Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 + x_2$. (A, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let C be an oriented plane curve. Prove that there exists a global parametrization of C if and only if C is connected. (An, CO 3)

20. Let S be an n-surface in $\mathbb{R}^{n+1}$, let $\mathbb{X}$ be a smooth tangent vector field on S, and let $p \in S$. Prove that there exists an open interval I containing 0 and a parametrized curve $\alpha : I \to S$ such that

    1. $\alpha(0) = p$
    2. $\dot{\alpha}(t) = \mathbb{X}(\alpha(t))$ for all $t \in I$
    3. If $\beta : \tilde{I} \to S$ is any other parametrized curve in S satisfying (i) and (ii), then $\tilde{I} \subseteq I$ and $\beta(t) = \alpha(t)$ for all $t \in \tilde{I}$.

    (An, CO 1)

21. Show that if the spherical image of a connected n-surface S is a single point, then S is part or all of an n-plane. (E, CO 2)

22. Let S be an oriented n-surface in $\mathbb{R}^{n+1}$ and let $\mathbf{v}$ be a unit vector in $S_p,\ p \in S$. Then prove that

    1. There exists an open set $V \subset \mathbb{R}^{n+1}$ containing p such that $S \cap \mathrm{N}(\mathbf{v}) \cap V$ is a plane curve.
    2. The curvature at p of this curve is equal to the normal curvature $k(v)$.

    (An, CO 4)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Perceive the ideas of graphs and level sets, vector fields, the tangent space, surfaces, vector fields on surfaces, orientation.	A	2, 5, 13, 18, 20	11
CO 2	Explain the fundamentals of the Gauss map, geodesics, and parallel transport.	An	4, 6, 8, 12, 15, 21	12
CO 3	Summarize the ideas of the Weingarten map, curvature of plane curves, arc length and line integrals.	An	3, 10, 11, 14, 19	11
CO 4	Estimate curvature of surfaces	E	1, 9, 16, 17, 22	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;