Reg. No .... Name **25U567**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT7 : ALGEBRA**

(For Regular 2023 Admission and Supplementary 2022/ 2021/ 2020/ 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define orbits of a permutation.
2. How many different commutative binary operations can be defined on a set of n elements?
3. Find the orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.
4. Define direct product of groups.
5. Show that if R is a ring, then for any $a, b \in R,\ (-a)(-b) = ab$.
6. Find the number of homomorphism from $\mathbb{Z}_n$ onto $\mathbb{Z}$.
7. True or false: every skew field is a field. Justify.
8. Find the quotient and reminder when -38 is divided by 7.
9. A cyclic group has a unique generator. True or false. Justify.
10. Define homomorphism of groups.
11. Define division ring.
12. Show that the determinant function is a homomorphism on $GL(n, \mathbb{R})$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Show that $*$ defined on $\mathbb{Q}^+$ by $a * b = ab/2$ makes $\mathbb{Q}^+$ an abelian group.
14. Prove that the group homomorphism $\phi : G \to G'$ is a one to one map if and only if Ker$(\phi) = \{e\}$.
15. Show that if p is a prime, then $\mathbb{Z}_p$ is a field.
16. Let σ be a permutation of a set A. Show that the relation defined, for $a, b \in A,\ a\mathbf{R}b$ if and only if $b = \sigma^n(a)$ for some $n \in \mathbb{Z}$, is an equivalence relation.
17. Show by an example that if R is not even an integral domain, it is still possible for R/N to be a field, where N is some ideal of R.
18. Let $\phi : G \to G'$ be a homomorphism. Show that if K' is a subgroup of G', then $\phi^{-1}[K']$ is a subgroup of G.
19. Draw the subgroup diagram of Klein 4 group.
20. Write all the elements of D_4.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Show that the commutator of group is a normal subgroup. Show that if N is a normal subgroup of G, then G/N is abelian if and only if C ⊆ N.

22. Show that a nonempty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H, \forall a, b \in H$

23. Prove that if $n \geq 2$, then the collection of all even permutations of $\{1, 2, 3, \ldots, n\}$ forms a subgroup of order $n!/2$ of the symmetric group S_n.

24. Let R be a ring with $a^2 = a, \ \ \forall a \in R$. Prove that R is a commutative ring.

(10 x 3 = 30)