Reg. No .... Name **25P160**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2025

SEMESTER 1 : MATHEMATICS

COURSE : 24P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS

(For Regular 2025 Admission & Improvement/Supplementary 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. If S is defined by the rectangle $|x| \leq a, |y| \leq b$, show that $f(x,y) = x^2 + y^2$ satisfies the Lipschitz condition. (U)
2. Show that the function $f(x,y) = y^{2/3}$ does not satisfy the Lipschitz condition on the rectangle $R : |x| \leq 1, |y| \leq 1$. (A)
3. Define orthogonal and orthonormal functions. (R, CO 1)
4. State sturm Seperation theorem. (R)
5. Find the critical points of the system $\frac{dx}{dt} = y^2 - 5x + 6, \frac{dy}{dt} = x - y$. (U)
6. Determine the nature and stability properties of the critical point (0, 0) for the linear autonomous system $\frac{dx}{dt} = -3x + 4y, \frac{dy}{dt} = -2x + 3y$. (A)
7. Write an equivalent system of first order equation for y''' = y'' - x^2(y')^2. (U)
8. Prove that P_n (-1) = (-1) ? (U)
9. Show that $|J_0(x)| \leq 1$. (A)
10. Calculate $(\frac{3}{2})!$ (U)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that $f(x,y) = xy$ satisfies a Lipschitz condition on any rectangle $a \leq x \leq b, c \leq y \leq d$. (A)
12. Let $y(x)$ be a nontrivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a,b]$.Then prove that $y(x)$ has atmost a finite number of zeros in this interval. (An, CO 1)
13. Legendre's equation can be written in the form $\frac{d}{dx}\left((1-x^2)y'\right) + n\left(n+1\right)y = 0$.Use this to prove that $\int_{-1}^{1} P_m\left(x\right)P_n\left(x\right)dx = 0,\ m \neq n$. (A)
14. Solve the vibrating string problem with initial shape (A, CO 1)

$$f(x) = \begin{cases} 2cx, & 0 \leq x \leq \frac{\pi}{2} \\ 2c(\pi - \frac{x}{\pi}), & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

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15. Show that for $n = 1, 2, 3....., |Jn(x)| \leq 1/2$. (E)
16. Show that $\frac{d}{dx}(x^p J_p(x)) = x^p J_{p-1}(x)$. (A)

17. Show that if the roots of the auxillary equation are real, distinct and of the opposite signs for a linear autonomous system , then the critical point is a saddle point. (An)

18. Explain the method of successive approximations system of two equations. (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. For the following linear system , find the general solution , differential equation of the paths and its solution. Sketch a few paths showing the direction of increasing t and discuss the stability of the critical point (0,0). (An)
dx/dt = x
dy/dt = -x+2y

20. State and prove an existence and uniqueness theorem for initial value problem. (E)

21. State and prove Rodrigues formula. (E, CO 1)

22. Explain the orthogonality property of Sturm Liouville Problem. ()

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Summarize the concepts of Sturm Separation theorem and Sturm Liouville problems	A	3, 12, 14, 21	10

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;