

**M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2025****SEMESTER 1: COMPUTER SCIENCE (ARTIFICIAL INTELLIGENCE)****COURSE: 24P1CAIT04: MATHEMATICS FOR COMPUTATIONAL INTELLIGENCE***(For Regular 2025 Admission & Improvement/Supplementary 2024 Admission)*

Time: 3 Hours

Max. Weightage: 30

Qn. No.	Question	CO No.	Level
<b>PART A</b>			
<i>(Answer any 8 questions. Each question carries 1 weightage)</i>			
1.	Define a vector space. Give an example.	1	U
2.	Define rank of a matrix. Find the rank of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ .	1	R
3.	Evaluate $l_1, l_2, l_\infty$ - norms of the vector $v = (-2, 3, 4)$ .	2	E
4.	Find the length of $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ .	2	E
5.	Compute the determinant and trace of $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 2 & 1 & 0 \end{bmatrix}$ .	3	E
6.	Let $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ . Find $ D , D^2$ and $D^{-1}$ .	3	R
7.	Find the difference quotient of $f(x) = \sqrt{x}$ .	4	E
8.	Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^2y + 3xy^2$ .	4	E
9.	Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$ .	4	E
10.	Check whether $f(x) = x^2 + 4x + 5$ is convex. Find Minimum of $f(x)$ .	5	An

**PART B**

*(Answer any 6 questions. Each question carries 2 weightage)*

11.	Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$ using elementary row operations.	1	E
12.	Check whether the vectors $u = (3, -1, 2)$ , $v = (2, 1, 1)$ in $\mathbb{R}^3$ are orthogonal. If not, find the angle between them.	2	An
13.	Write the Cholesky decomposition of $A = \begin{bmatrix} 25 & 15 \\ 15 & 18 \end{bmatrix}$ .	3	E
14.	Check whether $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is diagonalizable. If yes, determine their diagonalizes form and a basis with respect to which the transformation matrices are diagonal. If no, give reasons for why it is not diagonalizable.	3	An
15.	Define a Maclaurin series. Find the Maclaurin series expansion for $f(x) = \sin(x) + \cos(x)$ .	4	A
16.	Find the Jacobian matrix of $F(x, y, z) = \begin{bmatrix} x + y + z \\ xy \\ yz \end{bmatrix}$ .	4	E
17.	Minimize $f(x) = x^2 + xy + y^2$ by Gradient descent optimization.	5	A
18.	Find a global minimum for $f(x, y) = x^4 + y^4$ .	5	E

**PART C**

*(Answer any 2 questions. Each question carries 5 weightage)*

19.	Consider the system: $x + y + z = 6$ , $2x + 3y + z = 10$ , $x + 2y + 3z = 13$ . a) Write the matrix representation of the system. b) Write the augmented matrix of the system. c) Solve the system of linear equations by using Gauss elimination method.	2	A
20.	a) Define an orthogonal basis. b) Check whether $x = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ , $y = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ form an orthogonal basis for $\mathbb{R}^2$ .	2	A

	<p>c) Let <math>x = [1 \ 1]^T, y = [2 \ 1]^T \in \mathbb{R}^2</math> and <math>\langle x, y \rangle = x^T \begin{bmatrix} 1 &amp; -1/2 \\ -1/2 &amp; 1 \end{bmatrix} y = x_1 y_1 - \frac{1}{2}(x_1 y_2 + x_2 y_1) + x_2 y_2</math>. Find the length of <math>x</math> and <math>y</math>.</p> <p>d) Find the distance between <math>x</math> and <math>y</math>.</p>		
21.	<p>a) Determine the eigen values and corresponding eigen vectors of <math>\begin{bmatrix} 4 &amp; 1 \\ 2 &amp; 3 \end{bmatrix}</math>.</p> <p>b) Write the multiplicities of each eigen value.</p> <p>c) Write the eigen decomposition of the given matrix.</p>	3	A
22.	<p>a) Define the terms: Lagrange multiplier and Lagrangian function.</p> <p>b) Find the maximum and minimum of <math>f(x, y) = x^2 + y^2</math> subject to the constraint <math>x + y = 10</math>.</p>	5	A

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