

**END SEMESTER EXAMINATION - NOVEMBER 2025****SEMESTER 1 : INTEGRATED M.Sc. PROGRAMME COMPUTER SCIENCE - DATA SCIENCE****COURSE : 21UP1CPCMT1 : GRAPH THEORY AND OPERATION RESEARCH***(For Regular 2025 Admission and Improvement/ Supplementary 2024/2023/2022/ 2021 Admissions)*

Time : Three Hours

Max. Weights: 30

**PART A****Answer any 8 questions****Weight: 1**

1. Convert the following LP problem in to a standard LP problem by adding Slack, Surplus Or Artificial variables.

$$\text{maximize: } z = 3x_1 + 9x_2$$

$$\text{subject to: } x_1 + 4x_2 \leq 8$$

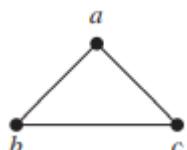
$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

2. Define connected components in a graph.

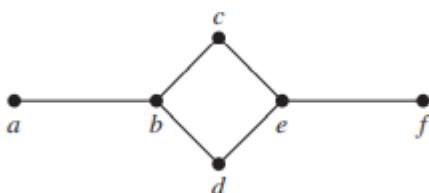
3. What is the value of the postfix expression  $723 * - 4 \uparrow 9 3 / + ?$

4. Draw all the spanning trees of the given simple graph.



5. Define isomorphism in graphs.

6. Draw all the spanning trees of the given simple graphs.



7. Define Euler circuit and Euler paths in a graph.

8. Explain graphical method of solving an LP Problem.

9. How to identify the solution obtained in MODI method is optimal or not?

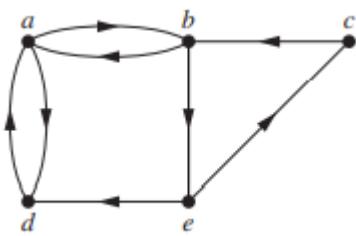
10. Define occupied cells and non-occupied cells.

**(1 x 8 = 8 weight)****PART B****Answer any 6 questions****Weights: 2**

11. Prove that a full m-ary tree with  $i$  internal vertices contain  $n = mi+1$  vertices.

12. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of these paths?

a) a,e,b,c,b      b) a,e,a,d,b,c,a  
c) e,b,a,d,b,e      d) c,b,d,a,e,c



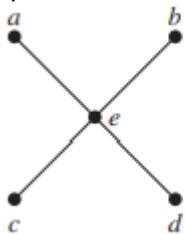
13. Draw these graphs.  
 a)  $K_5$       b)  $K_{1,5}$   
 c)  $C_5$       d)  $W_7$

14. A company has three production facilities  $S_1$ ,  $S_2$  and  $S_3$  with production capacity of 7, 9, and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  with requirement of 5, 6, 7 and 14 units (100s) per week, respectively. The transportation cost (in rupees) per unit between factories to warehouses are given in the table below: Use North-West Corner Method to find an initial basic feasible solution to the given transportation problem.

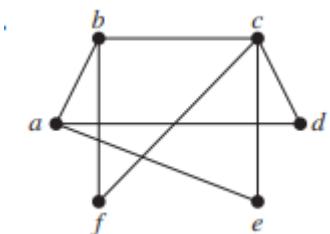
	$D_1$	$D_2$	$D_3$	$D_4$	Capacity
$S_1$	19	30	50	10	7
$S_2$	70	30	40	60	9
$S_3$	40	8	70	20	18
Demand	5	8	7	14	34

15. Determine whether these graphs are bipartite. If not, give reason.

a)



b)



16. Solve the following Linear programming problem graphically;

$$\text{minimize: } z = 3x_1 - x_2$$

$$\text{subject to: } x_1 - 2x_2 \geq 4$$

$$x_1 + x_2 \leq 8$$

$$-4x_1 + 2x_2 \leq 20$$

$$x_2 \leq 4$$

$$x_1 \leq 8$$

$$x_1 \geq 4$$

$$x_1, x_2 \geq 0$$

17. Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

18. Use backtracking to find a subset, if it exists, of the set  $\{27, 24, 19, 14, 11, 8\}$  with sum  
 a) 60.      b) 41.

**(2 x 6 = 12 weight)**

**PART C**

**Answer any 2 questions**

**Weights: 5**

19. A company has factories at  $F_1, F_2$ , and  $F_3$  which supply to warehouses at  $W_1, W_2$  and  $W_3$ . Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirement are 180, 120 and 150 units, respectively. Unit shipping cost (in rupees) are as follows:

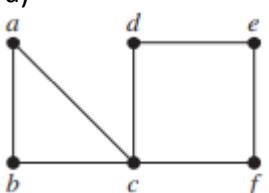
	$W_1$	$W_2$	$W_3$	Supply
$F_1$	16	20	12	200
$F_2$	14	8	18	160
$F_3$	26	24	16	90
Demand	180	120	150	450

Determine the optimal distribution for this company to minimize total shipping cost.

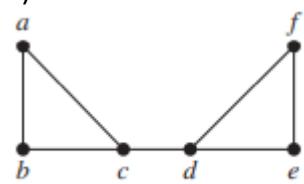
20. Use Big-M Method to solve the following linear programming problem;  
 maximize:  $z = 5x_1 + x_2$   
 subject to:  $5x_1 + 2x_2 \leq 20$   
 $x_1 \geq 3$   
 $x_2 \leq 5$   
 $x_1, x_2 \geq 0$

21. Show that in every simple graph there is a path from any vertex of odd degree. Find all cut vertices and cut edges of the given graphs.

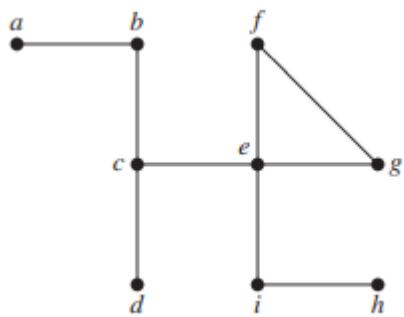
a)



b)



c)



22. a) Represent the expressions  $(x + xy) + (x/y)$  and  $x + ((xy + x)/y)$  using binary trees.

Write these expressions in

b) prefix notation.

c) postfix notation.

d) infix notation.

**(5 x 2 = 10 weight)**
