Reg. No .... Name **25P144**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2025

SEMESTER 1 : MATHEMATICS

COURSE : **24P1MATT03 : REAL ANALYSIS**

(For Regular 2025 Admission & Improvement/ Supplementary 2024 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Show that the power series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ has infinite radius of convergence.	(A)
2.	If $a < S < b$, f is bounded on $[a, b]$, f is continuous at S, and $\alpha(x) = I(x - S)$, prove that $\int\limits_a^b f d\alpha = f(S)$.	()
3.	Prove or disprove: there exists a function of bounded variation on $[a, b]$ whose derivative is not bounded on (a, b).	(An)
4.	Show by an example that $\lim_{m\to\infty} \lim_{n\to\infty} S_{m,n} \neq \lim_{n\to\infty} \lim_{m\to\infty} S_{m,n}$	(An, CO 2)
5.	Show that a polynomial is always a function of bounded variation on every compact interval.	(A)
6.	Show that the power series $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$ has radius of convergence equal to 1	(A)
7.	Suppose $f \geq 0$, f is continuous on [a,b] and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0,\ \forall x \in [a, b]$.	(R)
8.	Prove or disprove "A convergent series of continuous functions may have a discontinuous sum"	(A)
9.	Show that $lim_{x\to 0} \frac{log(1+x)}{x} = 1$	(A)
10.	Show by an example that a function of bounded variation need not be continuous.	(E)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Let $f_n(x) = \dfrac{x}{1 + nx^2}, \text{for n} = 1, 2, 3.., x \in \mathbb{R}$. Prove that the sequence $\{f_n(x)\}$ uniformly converges to some f on $\mathbb{R}$.	(A)
12.	Discuss the radius of convergence of the series $\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \ldots\ldots$	(A)
13.	Define equivalent paths. State and prove a necessary and sufficient condition for equivalence of two paths which are one to one on its domain. Give an example for non-equivalent paths.	(U)
14.	Evaluate $\int_0^1 x\, d[x]$	(A)
15.	Show that the sequence $\{f_n(x)\}$ where $f_n(x) = x^{n-1}(1 - x)$ uniformly convergent on $[0, 1]$	(A)

16. Let **f**:[a,b]$\to \mathbb{R}^n$ be a path, where **f**=$(f_1, f_2 \ldots, f_n)$. Prove that **f** is rectifiable if and only if each component f_k is of bounded variation on [a,b]. (U)

17. Evaluate $\lim_{x\to 0} \frac{e-(1+x)^{1/x}}{x}$ (A)

18. Show that f∈R(α) on [a, b] if and only if for every ε>0 there exists a partition P such that U(P,f,α)-L(P,f,α)<ε (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. a) If $f \in \mathscr{R}(\alpha)$ then show that $cf \in \mathscr{R}(\alpha)$ for every constant c and $\int_a^b cf d\alpha = c\int_a^b f d\alpha$ (A)
b) If $f_1(x) \le f_2(x)$ for all x in $[a,b]$, then show that $\int_a^b f_1 d\alpha \le \int_a^b f_2 d\alpha$

20. Discuss about the exponential and logarithmic functions (An)

21. Suppose f_n is a sequence of functions, differentiable on $[a,b]$ and such that $f_n(x_0)$ converges for some point x_0 on $[a,b]$. If $f_n^{'}$ converges uniformly on $[a,b]$, then prove that f_n converges uniformly on [a, b], to a function f, and **f** $'(x) = \lim_{n\to\infty} f_n'(x), (a \le x \le b)$ (R)

22. Let f be of bounded variation on [a,b]. For $x \in (a,b]$,let $V(x) = V_f(a,x)$ and put $V(a) = 0$. Then prove that every point of continuity of f is also a point of continuity of V. Prove also that the converse is true. (An)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 2	Illustrate the properties of Riemann-Stieljes integral.	U	4	1

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;