Reg. No .... Name **25P128**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2025

SEMESTER 1 : MATHEMATICS

COURSE : **24P1MATT02 : ALGEBRA - I**

(For Regular - 2025 Admission and Improvement/ Supplementary 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Explain the terms elementary contractions and reduced words. (U, CO 3)
2. Determine the number of abelian groups (upto isomorphism) of order 10^5. (A, CO 1)
3. Find the order of $(3, 6, 12, 16)$ in the group $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{20} \times \mathbb{Z}_{24}$. (A, CO 1)
4. Define a Sylow p-subgroup of a group G. Give an example. (R)
5. When are two free groups said to be isomorphic? (A, CO 3)
6. Prove that a group of order 81 is solvable. (An, CO 2)
7. Is every normal series of a group G a subnormal series of G? Justify your answer. (A, CO 2)
8. How many elements of finite order does $\mathbb{Z}_2 \times \mathbb{Z} \times \mathbb{Z}_4$ have? (A, CO 1)
9. What is the order of a Sylow 3-subgroup of a group of order (A, CO 4)
   (a) 12? (b) 54? (c)3?
10. Is the normalizer in a group G of a subgroup H of G always a normal subgroup of G? Justify your answer. (E, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find all abelian groups up to isomorphism of order 16. (An, CO 1)
12. How many abelian groups up to isomorphism are there of order 15? How many non abelian groups upto isomorphism are there of order 15?Justify your answer. (A, CO 4)
13. In the group $\mathbb{Z}_{24}$, let $H = < 4 >$ and $N = < 6 >$. List the elements in HN and in $H \cap N$. Further list the cosets in HN/N and in $H/(H \cap N)$ showing the elements in each coset. Give the correspondence between HN/N and $H/(H \cap N)$ as described in the second isomorphism theorem. (E, CO 2)
14. Define a decomposable group. Give an example. Show that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime. (U, CO 1)
15. Show that the group with presentation $(a, b : a^n = b^2 = abab = 1)$ is isomorphic to the dihedral group D_n. (E, CO 3)
16. Let $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_{12}$ be the homomorphism such that $\phi(1) = 2$. Find the Kernel K of ϕ. List the cosets in $\mathbb{Z}_{18}/K$ showing the elements in each coset. Find the group $\phi[\mathbb{Z}_{18}]$. Give the correspondence between Z_{18}/K and $\mathbb{Z}_{12}$ as described in the first isomorphism theorem. (E, CO 2)
17. Show that every group G is a homomorphic image of a free group. (An, CO 3)
18. Show that for a prime number p, every group of order p^2 is abelian. (E, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Prove that all simple groups of order at most 20 are cyclic. (E, CO 4)

20. Show that every group of order 1645 is cyclic. (E, CO 4)

21. State and prove a necessary and sufficient condition for the group $\mathbb{Z}_m \times \mathbb{Z}_n$ to be cyclic. (An, CO 1)

22. Let $G \neq \{0\}$ be a free abelian group with a finite basis. Show that every basis of G is finite and all bases of G have the same number of elements. (E)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze groups using generating sets, direct products, finitely generated abelian groups, and group action on a set.	E	2, 3, 8, 11, 14, 21	12
CO 2	Comprehend Isomorphism Theorems, subnormal/normal series, and solvable groups.	E	6, 7, 13, 16	6
CO 3	Apply free groups and free abelian groups in the proof of the fundamental theorem of abelian groups and in group presentations.	E	1, 5, 15, 17	6
CO 4	Comprehend Sylow theorems and apply the Sylow theory to study groups of different orders.	E	9, 10, 12, 18, 19, 20	16

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;