Reg. No .... Name **25P112**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2025

SEMESTER 1 : MATHEMATICS

COURSE : **24P1MATT01 : LINEAR ALGEBRA**

(For Regular - 2025 Admission and Improvement/Supplementary 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Define annihilator of a subset S of a vector space V. What is the annihilator of $S = \{0\}$? (U)
2. Define sign of a permutation (R, CO 3)
3. Let D be a 2-linear function with the property that $D(A) = 0$ for all $n \times n$ matrices A over K having equal rows. Show that D is alternating. (A, CO 3)
4. Find three vectors in $\mathbb{R}^3$ which are linearly dependent, and are such that any two of them are linearly independent. (An, CO 1)
5. Define alternating n-linear function. (U, CO 3)
6. Define minimal polynomial for a linear operator T on a finite dimensional vector space V. State three properties which characterize the minimal polynomial. (U)
7. Is there a linear transformation T from R^3 into R^2 such that $T(1,-1,1) = (1,0)$ and $T(1,1,1) = (0,1)$? Justify. (U)
8. Describe the range and null space of the differentiation transformation defined on the vector space of polynomials of degree less than or equal to n. (U)
9. If the characteristic polynomial of an operator is $x^4 - 2x^2 + 1$, what are the possible candidates for its minimal polynomial? Justify. (U)
10. Define a vector space. Is $\mathbb{R}$ a vector space over $\mathbb{C}$? (U, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Let T be a linear operator on $\mathbb{R}^2$ defined by $T(1,0) = (0,1)$ and $T(0,1) = (-1,0)$. Find the subspace of R^2 which is invariant under T. (A)
12. Let V and W be finite-dimensional vector spaces over the field $\mathbb{F}$. Prove that V and W are isomorphic if and only if $dimV = dimW$ (A)
13. Let V be the vector space over the complex numbers of all functions from R into C, i.e., the space of all complex-valued functions on the real line. Let $f_1(x) = 1, f_2(x) = e^{ix}, f_3(x) = e^{-ix}$. Prove that f_1, f_2 and f_3 are linearly independent. (A, CO 1)
14. If A and B are $n \times n$ matrices over the field F, show that trace (AB)= trace (BA). Hence show that similar matrices have the same trace. (A)
15. Let A be an $n \times n$ matrix over K. Prove that A is invertible over K if and only if $det\ A$ is invertible over K. (A, CO 3)
16. Let V be a finite dimensional vector space over a field $\mathbb{F}$ and T be a linear operator on V . Prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over $\mathbb{F}$. (A)
17. Verify whether $D(A) = A_{11} + A_{22} + A_{33}$ is $3-$linear. (A, CO 3)
18. Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \ldots, \beta_m$.Show that any independent set of vectors in V is finite and contains no more than m elements. (An, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $A = [a_{ij}] \in M_9(\mathbb{R})$ be defined by $a_{ij} = 1\ \forall i, j$. Find the characteristic and minimal polynomials for A. (An)

20. Let m and n be positive integers and let F be a field.Suppose W is a subspace of F^n and $dim\, W \leq m$.Show that there is precisely one $m \times n$ row reduced echelon matrix over F which has W as its row space. (An, CO 1)

21. (a) Find the determinant of A^{10} where $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & -25 \\ 0 & 0 & 1 \end{bmatrix}$. Justify your answer. (E, CO 3)

    (b) Show that a linear combination of n-linear functions is n-linear.

22. (a) Define rank and nullity of a linear transformation.

    (b) Let V be finite dimensional and T :V →W be a linear trasnformation. Prove that Rank T + nullity T = dim V. (U)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1		A	4, 10, 13, 18, 20	11
CO 3		An	2, 3, 5, 15, 17, 21	12

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;