25FYU343

Reg. No……………………………………….   Name…………………………………………..

B A, B SC, B COM DEGREE END SEMESTER EXAMINATION – OCTOBER 2025

UGP (HONS.) SEMESTER - 3: DISCIPLINE SPECIFIC COURSE

COURSE: 24UPHYDSC202 - ESSENTIALS OF MATHEMATICAL PHYSICS

(For Regular 2024 Admission)

Time: 2 Hours Max. Marks: 70

PART A (Short Answers)

2 marks each - Answer any 14 questions

1. Show that the scalar product of two perpendicular vectors is zero. (U, CO 1)
2. Find the angle between vectors $a = \hat{\imath} + \hat{\jmath}$ and $b = \hat{\imath} - \hat{\jmath}$. (A, CO 1)
3. If $a = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $b = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$, find $a \times b$. (A, CO 1)
4. What are the conditions for a set of vectors to form a basis in a vector space? (U, CO 1)
5. Find the trace of $B = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$. (A, CO 2, CO 7)
6. Solve the system:

$$x + y = 3$$
$$x - y = 1$$

(A, CO 2, CO 7)

7. Show that, $Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, is orthogonal. (A, CO 2, CO 7)
8. Find the gradient of the scalar field, $\varphi = x^3 + y^2 z - xyz^2$,
   at the point (1, -1, 2). (A, CO 4, CO 7)
9. Write the expressions for divergence and curl in both Cartesian and spherical polar coordinate systems. (A, CO 4, CO 5, CO 7)
10. Differentiate the vector function, $r(t) = t^2\,\hat{\imath} + \sin t\,\hat{\jmath} + e^t\,\hat{k}$. (A, CO 4, CO 7)
11. Find the curl of the vector field, $F = xyz\,\hat{\imath} + x^2 z\,\hat{\jmath} + y^2 x\,\hat{k}$, and
    evaluate it at (1, 1, 1). (A, CO 4, CO 5, CO 7)
12. State the divergence theorem and mention one physical application. (U, CO 6)
13. Distinguish between simply connected and doubly connected regions. (U, CO 6)
14. Determine whether the vector field, $F = y\,\hat{\imath} + x\,\hat{\jmath}$, is conservative. (A, CO 6, CO 7)
15. Evaluate the line integral, $\int x\,dx + y\,dy$, along the straight line from (0,0) to (1,1).
    (A, CO 6, CO 7)
16. Write the integral form of divergence of a vector field. (U, CO 6)

(14 x 2 = 28)

PART B

Short Essays or Problems (6 marks each), Answer any 7 questions

17. Derive the vector equation of a plane passing through a point P (1, 2, 3) and perpendicular to vector $n = 2\hat{\imath} - \hat{\jmath} + \hat{k}$. (A, CO 1)

18. Given vectors $a = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $b = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, and $c = -\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, compute the scalar triple product and interpret its physical meaning. (An, CO 1)

19. Find the inverse of, $A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$, using the adjoint method. (A, CO 2, CO 7)

20. Find the eigenvalues and eigenvectors of,

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$ (A, CO 3, CO 7)

21. Determine whether, $U = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$, is unitary. (A, CO 2, CO 7)

22. The vector field, $F = r\cos\theta\ \hat{r} + r\sin\theta\ \hat{\theta} + z\,\hat{z}$, is given in cylindrical polar coordinates.
    (a) Express this field in Cartesian coordinates.
    (b) Compute its divergence in both coordinate systems and verify consistency.
    (A, CO 4, CO 5, CO 7)

23. Let $F = yz\,\hat{\imath} + zx\,\hat{\jmath} + xy\,\hat{k}$. Find $\nabla . F$ and $\nabla \times F$. (A, CO 4, CO 5, CO 7)

24. Verify Stokes' theorem for the vector field, $A = y\,\hat{\imath} - x\,\hat{\jmath} + z\,\hat{k}$, over the hemispherical surface, $x^2 + y^2 + z^2 = a^2,\ z \geq 0$. (A, CO 6, CO 7)

25. Find the surface integral of, $F = xz\,\hat{\imath} + yz\,\hat{\jmath} + z^2\,\hat{k}$, over the top face of the cube bounded by $(0 \leq x, y, z \leq 1)$. (A, CO 6, CO 7)

(6 x 7 = 42)

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