B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025
SEMESTER 3 : COMPUTER APPLICATIONS
COURSE : **19U3CRCMT3 : CALCULUS**
(For Improvement/Supplementary 2023/2022/2021/2020/2019 Admissions)

Time : Three Hours   Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Evaluate the integral $\int_0^{\frac{\pi}{3}} \frac{\tan\theta \, d\theta}{\sqrt{2 \sec\theta}}$.

2. Use the chain rule to find the derivative of $w = x^2 + y^2$ with respect to t along the path x = cos t , y = sin t. What is the derivative's value at $t = \pi$.

3. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$.

4. Find the radius of curvature at the point θ on the curve x=a(θ-sin θ), y=a(1-cos θ).

5. Evaluate $\iint_R xy \, dx \, dy$, where R is the region $x^2 + y^2 \le a^2, \ x \ge 0, \ y \ge 0$.

6. Expand ln (x+a) in powers of x, using Tayor's series.

7. Find $f_x, \ f_y$ and f_z if $f(x, \ y, \ z) = e^{-(x^2+y^2+z^2)}$.

8. If $y = e^{a \sin^{-1} x}$, prove that $\left(1 - x^2\right)y_2 - xy_1 - a^2 y = 0$.

9. Find the Jacobian J(u,v) for the transformation x = u cosv, y = u sinv .

10. The region between the curve y=$2\sqrt{x}$, $0 \le x \le 2$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume using Disk method.

11. Find the length of the curve $x = \cos^3 t, \ y = \sin^3 t, \ \ 0 \le t \le 2\pi$.

12. Evaluate $\iint_R y \, dy \, dx$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

                                                  (2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the centre of curvature of the curve $x = a \ \cos^3 \theta, \ y = a \sin^3 \theta$.

14. Find the ranges of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also determine the points of inflection and the inflectional tangents to the curves.

15. Find by double integration, the area of the region enclosed by the lemniscate $r^2 = a^2 \cos 2\theta$.

16. Evaluate the cylindrical co-ordinate integral $\int_0^{2\pi} \int_0^1 \int_r^{1/\sqrt{2-r^2}} 3 \, dz \, rdr \, d\theta$.

17. Find the length of the curve y = log sec x between points given by x = 0 and x = $\frac{\pi}{3}$

18. Find all local maxima, local minima and saddle points of the function $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

19. The region bounded by the curve $y = \sqrt{4x - x^2}$, the x-axis and the line x=2 is revolved about x-axis to generate a solid. Find the volume of the solid by shell method.

20. If $u = \sin^{-1}\left(\frac{x-y}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

                                                  (5 x 5 = 25)

21. a) Find the surface area of the cone frustrum generated by revolving the line segment $y = \frac{x}{2} + \frac{1}{2}$, $1 \le x \le 3$, about the x-axis.

    b) Using washer method, find the volume of the solid generated by revolving the region in the first quadrant bounded by the circle $x^2 + y^2 = 3$ and the lines $x = \sqrt{3} \ and \ y = \sqrt{3} \ about \ y - axis$.

22. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

23. Find the volume of the region bounded below by the plane z = 0, laterally by the cylinder $x^2 + y^2 = 1$ and above by the paraboloid $z = x^2 + y^2$.

24. Using Lagrange multipliers, find the minimum value of x+y, subject to the constraints xy=16 , x>0, y>0.

(10 x 3 = 30)