Reg. No .................... Name .............................. **25U327-S**

**B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025**

**SEMESTER 3 : MATHEMATICS**

COURSE : **19U3CRMAT03 : VECTOR CALCULUS, THEORY OF EQUATIONS AND MATRICES**

*(For Improvement/Supplementary 2023/ 2022/2021/2020/2019 Admissions)*

Time : Three Hours Max. Marks: 75

**PART A**

**Answer any 10 (2 marks each)**

1. Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.
2. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.
3. State Cayley Hamilton theorem and verify it for A = $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.
4. Form a rational quartic equation whose roots are 1,-1 and $2+\sqrt{3}$
5. State Stoke's theorem.
6. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & -1 \\ -1 & -2 & 6 \end{bmatrix}$.
7. Find the sum of the cubes of the roots of the equation $x^3 - 2x^2 + x + 1 = 0$
8. Define eigen value and eigen vector.
9. If $\bar{r} = ti - t^2 j + (t-1)k \; and \; \bar{s} = 2ti + 6kt, Evaluate \; \int_0^2 \bar{r} \times \bar{s} dt$.
10. Show that $\nabla\left(\overline{A} \times \overline{B}\right) = \left(\nabla.\overline{B}\right)\overline{A} - \left(\nabla.\overline{A}\right)\overline{B} + \left(\overline{B}.\nabla\right)\overline{A} - \left(\overline{A}.\nabla\right)\overline{B}.$
11. Form a rational quartic equation whose roots are $2, -1$ and $1-\sqrt{3}$.
12. Evaluate the angle between the normals to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3).

**(2 x 10 = 20)**

**PART B**

**Answer any 5 (5 marks each)**

13. If $\overline{A} = \left(3xz^2\right)i - (yz)j + (x+2z)k. Find \; curl \left(curl \overline{A}\right)$.
14. Use Gauss Jordan method to find the inverse of $\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix}$.
15. If $\overline{F} = (2x^2 - 3z)i - 2xyj - 4xk$, then evaluate $\iiint_V \nabla$ X $\overline{F}\, dv$, where V is the closed region bounded by the planes $x=0, y=0, z=0$ and $2x+2y+z=4$.
16. Solve the equation $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$
17. If $\bar{r} = xi + yi + zk \; show \; that \; grad \left(\frac{1}{r}\right) = \frac{-\bar{r}}{r^3}$.

18. Evaluate $\int_c \left(2x^2y + y + z^2\right)i + 2\left(1 + yz^2\right)j + \left(2z + 3y^2z^2\right)k.\, d\bar{r}$ along the curve $c : y^2 + z^2 = a^2, x = 0$.

19. Reduce the matrix A = $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ into normal form and find the rank .

20. Solve $4x^4 - 85x^3 + 357x^2 - 340x + 64 = 0$ given that its roots are in geometric progression.

**(5 x 5 = 25)**

**PART C**

**Answer any 3 (10 marks each)**

21. Solve $x^4 - 9x^2 + 4x + 12 = 0$ given that it has repeated roots.

22. Evaluate $\iint_s \overline{A}.\, \hat{n} ds$, where $\overline{A} = (x + y^2)i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

23. Verify Cayley Hamilton theorem for the matrix A and find $A^{-1}$, where A = $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

24. Show that a) $\nabla\left(\bar{a} \,.\, \bar{u}\right) = (\bar{a} \,.\, \nabla)\bar{u} \; + \bar{a} \; \times (\nabla \times \bar{u})$
b)Prove that the vector $f\left(r\right)\bar{r}$ is irrotational.

**(10 x 3 = 30)**