Reg. No .... Name **25U528**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT05 : REAL ANALYSIS - I**

(For Regular 2023 Admission and Supplementary 2022/ 2021/ 2020/ 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define a strictly decreasing sequence and give an example.
2. State D'Alembert's ratio test for convergence of a series.
3. State Raabe's test for convergence of a series.
4. State Cesaro's theorem.
5. Prove that $\lim_{x\to 0} \log|x| = -\infty$.
6. Prove that $|-x| = |x|$.
7. Define a conditionally convergent series and give an example.
8. State the order completeness property.
9. State Cauchy's first theorem on limits.
10. Show that a finite set has no limit points.
11. Define limit point of a sequence. What are the limit points of the sequence $\{(-1)^n\}$?
12. Define closure of a set. Explain the concept by means of an example.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. If $\sum u_n$ is a positive term series such that $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) > 1$, prove that $\sum u_n$ is convergent.
14. Prove that $|x - y| \geq \big||x| - |y|\big|$ for all real numbers x and y.
15. Show that the sequence $\{S_n\}$ where $S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{(n-1)!}, \ \forall n \in \mathbb{N}$ is convergent.
16. Test the convergence of the series $x + 2x^2 + 3x^3 + 4x^4 + \cdots$
17. Show that the infimum m of a bounded non empty set $S \subseteq \mathbb{R}$ always belongs to the closure of S.
18. Show that $\lim_{x\to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist.
19. Discuss the convergence of the series $\frac{1}{2} + \frac{2}{3}x + \frac{3}{4}^2 x^2 + \frac{4}{5}^3 x^3 + \cdots$, where $x > 0$.
20. Prove that a monotonic increasing sequence which is not bounded above diverges to $+\infty$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (a) If $f : A \to B$ is one to one and B is countable, then prove that A is countable.
    (b) Show that every subset of a countable set is countable.
22. State and prove Leibnitz's test for convergence of an alternating series.
23. State the logarithmic test for convergence of a positive term series . Discuss the convergence of the series $1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \dots$ for $x > 0$.
24. State and prove the Bolzano Weierstrass theorem for sequences. Is the converse true? Justify.

(10 x 3 = 30)