Reg. No .... Name **25P322**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 3 : MATHEMATICS

COURSE : **24P3MATT12 : FUNCTIONAL ANALYSIS - I**

(For Regular - 2024 Admission)

Time: Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Let X be a normed linear space and $P : X \to X$ be a projection. Show that $I - P : X \to X$ is also a projection, where I is the identity map on X. (U, CO 3)
2. State the Banach - Steinhaus theorem. (U, CO 3)
3. Define summable and absolutely summable series in a nls X and give examples for each. (U, CO 2)
4. Verify that the norms $||.||_1$ and $||.||_\infty$ on K^2 are equivalent. (A, CO 1)
5. Let X be the linear space of all Riemann integrable functions on $[a, b]$ with norm $||x|| = \int_a^b x(t)\,dt$. What is the completion of X? (An, CO 4)
6. When are two normed linear spaces said to be linearly homeomorphic? When are two normed linear spaces said to be linearly isometric? (U, CO 1)
7. State the Hahn Banach Extension theorem. (U, CO 2)
8. Explain the completion X_c of a $nls\ X$. (U, CO 4)
9. Is the closed unit ball in an infinite dimensional space compact? Justify your answer. (An, CO 1)
10. Show that every finite dimensional nls is a Banach space. (A, CO 2)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. State and prove Jensen's inequality. (An, CO 1)
12. Let X be a normed linear space and P be a projection on X. If P is continuous, show that $R(P)$ and $Z(P)$ are closed in X. (An, CO 3)
13. Let X be a normed linear space. If E is convex in X, prove that E° and $\overline{E}$ are convex. (A, CO 1)
14. Show that c with $||.||_\infty$ is a Banach space. (An, CO 2)
15. Let Y and Z be closed subspaces of a Banach space X with the property that $Y + Z = X$ and $Y \cap Z = \{0\}$. For $x \in X$, suppose $x = y + z$, where $y \in Y$ and $z \in Z$. Define $P : X \to X$ by $P(x) = y$. Show that P is continuous. (A, CO 2)
16. If an operator A is of finite rank on a linear space, show that $A - I$ is one-one if and only if $A - I$ is onto. (An, CO 3)
17. Does a convergent sequence of finite rank operators on $BL(X)$, where X is a Banach space, always have a finite rank operator as its limit? Justify your answer. (An, CO 4)
18. Let X and Y be normed linear spaces and $F \in BL(X, Y)$. Show that F' is one to one if and only if $R(F)$ is dense in Y. (A, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove the open mapping theorem. (E, CO 3)

20. State and prove the Hahn-Banach separation theorem. (E, CO 2)

21. Let X be a Banach space and $A \in BL(X)$. Prove that
(a) $e(A) \subset a(A) \subset s(A)$
(b) If E is a set of eigen vectors of A such that no two elements of E correspond to the same eigen value of A, then E is linearly independent. (E, CO 4)

22. Define l^p, $1 \leq p \leq \infty$. What is the usual norm on l^p? Verify that l^p is a normed linear space with respect to this norm. (An, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze normed linear spaces and the continuity of linear maps.	An	4, 6, 9, 11, 13, 22	12
CO 2	Analyze the Hahn Banach Theorems and banach spaces	An	3, 7, 10, 14, 15, 20	12
CO 3	Analyze the uniform boundedness theorem, the closed graph theorem and the open mapping theorem.	E	1, 2, 12, 16, 19	11
CO 4	Analyze the spectrum of a bounded operator	E	5, 8, 17, 18, 21	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;