Reg. No .... Name **25P335**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 3 : MATHEMATICS

COURSE : **24P3MATT13 : ADVANCED TOPOLOGY**

(For Regular - 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Prove that the evaluation function of a family of functions $\{f_i\}$ is continuous if each f_i is continuous.	(U, CO 2)
2.	State Embedding Lemma.	(R, CO 2)
3.	Define a subnet of a net S in X.	(R, CO 3)
4.	Prove that T_0- axiom is a productive property.	()
5.	Define a filter on a set X.	(R, CO 3)
6.	Let S be a sub-base for a topological space X. Prove that if X is completely regular, then for each $V \in S$ and for each $x \in V$, there exist a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$.	()
7.	Prove that T_2 axiom is a productive property.	()
8.	Define a filter on a set X.	(R, CO 3)
9.	Define one point compactification of the space (X, T).	(U, CO 4)
10.	In a second countable space prove that countable compactness implies compactness	(A, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Let $f_1, f_2, f_3 : \mathbb{R} \to \mathbb{R}$ be defined by $f_1(x) = \cos x$, $f_2(x) = \sin x$, $f_3(x) = x$ for $x \in \mathbb{R}$. Describe the evaluation maps of the families $\{f_1, f_2\}$; $\{f_1, f_3\}$ and $\{f_1, f_2, f_3\}$. Which of these families distinguish points?	(A, CO 2)
12.	If a space is second countable and T_3, prove that it is embeddable in the Hilbert cube.	(U, CO 2)
13.	Let X be the topological product of an indexed family of spaces, $\{X_i; i \in I\}$. Let $\mathcal{F}$ be a filter on X and let $x \in X$. Prove that $\mathcal{F}$ converges to x in X iff for each $i \in I$, the image filter, $\pi_{i_\#}(\mathcal{F})$ converges to $\pi_i(x)$ in X_i.	(A, CO 3)
14.	Let τ be the product topology on the set πX_i where $\{(X_i, \tau_i); i \in I\}$ is an indexed collection of topological spaces. Prove that the family of all subsets of the form $\pi_i^{-1}(V_i)$ for $V_i \in \tau_i, i \in I$, is a sub-base for τ.Also prove that the family of all large boxes all of whose sides are open in the respective spaces, is a base for τ.	()
15.	Prove that a metric space is compact if it is countably compact.	(U, CO 4)
16.	Let X, Y be sets, $f : X \to Y$, a function and $\mathcal{F}$ a filter on X. Prove that the family $f(\mathcal{F}) = \{f(A)/A \in \mathcal{F}\}$ is a base for a filter on Y. Define the image filter of $\mathcal{F}$ under f.	(A, CO 3)

17. Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Then prove that X is normal. ()

18. If X is Hausdorff and locally compact at a point x in X, prove that the family of compact neighbourhood of x is a local base at x. (U, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove Tietze extension theorems. (U, CO 1)

20. Define an ultra filter on a set X and prove that every filter is contained in an ultra filter. (U, CO 3)

21. Let (X^+, τ^+) be one point compactification of the space (X, τ). Prove that (i) τ^+/X is the topology τ on X (ii) The space (X^+, τ^+) is compact (iii) X is dense in X^+ iff X is not compact. (A, CO 4)

22. State and prove Urysohn Embedding Theorem. (U, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1		U	19	5
CO 2		U	1, 2, 11, 12, 22	11
CO 3		U	3, 5, 8, 13, 16, 20	14
CO 4		A	9, 10, 15, 18, 21	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;