Reg. No .... Name **25P347**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 3 : MATHEMATICS

COURSE : **24P3MATT14 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2024 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	State Schwarz reflection principle.	(R)
2.	State Mittag-Leffler's theorem.	(R)
3.	State Hadamard factorization theorem.	(E)
4.	FInd the order of the functions $a) \sin(z) \quad b) cosh\sqrt{z}$	(A)
5.	State Bohr- Mollerup theorem by defining the Gamma function.	(An)
6.	State a new version of Cauchy's Integral formula.	(R)
7.	State Riemann Mapping Theorem.	(U)
8.	Find the order of $f(z) = e^{e^z}$	(A)
9.	If f(z) and if(z) are analytic in a region D, then show that f(z) is constant in that region D	(A)
10.	Show that C and $D = \{z : \|z\| < 1\}$ are homeomorphic by defining a homoemorphism.	(A)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Let $Rez_n \geq 0$ for all$n \geq 1$. Then show that $\prod_{k=1}^{\infty} z_n$ converges to a non zero number iff the series$\sum_{n=1}^{\infty} logz_n$ converges.	(An)
12.	State and prove the Weierstrass factorization theorem.	(U)
13.	Let f be an analytic function on a region containing $B(\bar{0}, r)$ and suppose that $a_1, \ldots, a_n$ are the zeros of f in $B(0; r)$ repeated according to multiplicity. If $f(0) \neq 0$ then $\log\|f(0)\| = -\sum_{k=1}^{n} \log\left(\frac{r}{\|a_k\|}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log\|f(re^{i\theta}\|d\theta$	(R)
14.	Let G be an open connected subset of C. If for any f in $H(G)$ such that $f(z) \neq 0$ for all z in G there is a function g in$H(G)$ such that $f(z) = [g(z)]^2$, then show that If $u : G \longrightarrow R$ is harmonic then there is a harmonic function $v : G \longrightarrow R$ such that $f = u + iv$ is analytic on G	(U)
15.	Show that a set $\mathscr{F} \subset C(G, \Omega)$ is normal iff for every compact set $K \subset G$ and $\delta > 0$ there are functions $f_1, f_2 \ldots . f_n$ in $\mathscr{F}$such that for f in $\mathscr{F}$, there is at least one $k, 1 \leq k \leq 1$, with $sup\{d(f(z), f_k(z)) : z \in K\} < \delta$.	(R)
16.	Show that $B(E)$ is a closed subalgebra of $C(K, \mathbb{C})$ that contains every rational function with a pole in E	(A)
17.	Let V and U be open subsets of $\mathbb{C}$ with $V \subset U$ and $\partial(V) \cap U = \phi$. If H is a component of U and $H \cap V \neq 0$ then show that $H \subset V$	(A)
18.	State and prove Euler's theorem	(U)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $\prod_{n=1}^{\infty} X_n, d)$, is a metric space. If $\xi^k = \{x_n^k\}_{n=1}^{\infty}$ is in $\prod_{n=1}^{\infty} X_n$, then prove that $\xi^k \longrightarrow \xi = \{x_n\}$ iff $x_n^k \longrightarrow x_n$ for each n. Also show that if each (X_n, d_n) is compact then X is compact. (R)

20. Show that $H(G)$ is a complete metric space (A)

21. If $a \in \mathbb{C} - K$ then show that $(z-a)^{-1} \in B(E)$ (R)

22. Let G be an open connected subset of (C). Then state and prove the equivalent conditions for G be simply connected. (An)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;