Reg. No .... Name **25P356**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2025

SEMESTER 3 : MATHEMATICS

COURSE : **24P3MATT15 : MULTIVARIATE CALCULUS AND FOURIER SERIES**

(For Regular - 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Find the number of basic k-forms in $\mathbb{R}^n$. (U, CO 4)
2. Define partial derivative. (R, CO 1)
3. Define saddle point. (R, CO 2)
4. Explain k - surface. (U, CO 4)
5. Explain the term Jacobian matrix. (A, CO 1)
6. Define stationary points. (R, CO 2)
7. Define the term primitive mapping. (An, CO 4)
8. Find the gradient vector $\nabla f(x, y, z)$ at the point $(1, 0, 1)$ of the function $f(x, y, z) = 3x^3y^4z^5$ (A, CO 1)
9. Evaluate $\int\limits_{-\pi}^{\pi} e^{inx}dx, n \in \mathbb{N}$ (A, CO 3)
10. Define Fourier series. (R, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find $J_f(r, \theta, \phi)$ where $f(r, \theta, \phi)$ is defined by $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$. (An, CO 4)
12. Show that $\sum\limits_{n=-N}^{N} e^{inx} = \dfrac{\sin(N+\frac{1}{2})x}{\sin(x/2)}$. (A, CO 3)
13. Assume f is differentiable at c with total derivative T_c. Then prove that the directional derivative $f'(c; u)$ exists for every u in $\mathbb{R}^n$ and we have $T_c(u) = f'(c; u)$. (A, CO 1)
14. Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$. (An)
15. Integrate $\int\limits_{I^2} e^{(x+y)}dxdy$ where $I^2 = [0, 1] \times [0, 2]$. (An, CO 4)
16. State and prove the localization theorem. (A, CO 3)
17. Let $f(x, y) = \begin{cases} x^2y^2\log(x^2+y^2) & if \quad (x, y) \neq (0, 0) \\ 0 & if \quad (x, y) = (0, 0) \end{cases}$. Compute the gradient vector $\nabla f(x, y)$ at those points(x, y)inR^2 where it exists. (A)
18. State and prove a global property of functions with a nonzero Jacobian determinant. (A, CO 2)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \leq M|t|$ for all $t \in (-\delta, \delta)$, then show that $\lim_{N \to \infty} S_N(f; x) = f(x)$. (An, CO 3)

20. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Then prove that the composite function $h = f \circ g$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$. (A, CO 1)

21. a)Suppose $w = \sum_I b_I(x) dx_I$ is the standard representation of a $k - form\ w$ in an open set $E \in R^n$. If $w = 0$ in E, then prove that $b_I(x) = 0$ for every increasing $k-$ index I and for every $x \in E$
b)Suppose T is a $1 - 1$ $\zeta^{'}-$ mapping of an open set $E \subset R^k$ into R^k such that $J_T(x) \neq 0\ \forall x \in E$. If f is a continuous function on R^k whose support is compact and lies in $T(E)$, then prove that $\int_{R^k} f(y)dy = \int_{R^k} f(T(x))|J_T(x)|dx$ (An, CO 4)

22. State and prove the mean value theorem for vector-valued functions. (An, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze Multivariable Differential Calculus The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule.	A	2, 5, 8, 13, 20	10
CO 2	Interpret Implicit functions and extremum problems, the mean value theorem for differentiable functions, a sufficient condition for differentiability	A	3, 6, 18, 22	9
CO 3	Explain Fourier series, trigonometric series and Parseval's formula and gamma function, stirling formula.	An	9, 10, 12, 16, 19	11
CO 4	Explain Integration of Differential Forms, primitive mappings, partitions of unity, change of variables, differential forms, Stokes theorem	An	1, 4, 7, 11, 15, 21	12

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;