Reg. No .... Name **25P4049**

MSc DEGREE END SEMESTER EXAMINATION- MARCH 2025

SEMESTER 4 : MATHEMATICS

COURSE : **21P4MATTEL20 : THEORY OF WAVELETS**

(For Regular - 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Suppose $M \in Z$and $z \in C$. When we say a sequence $\{z_n\}_{n=M}^{\infty}$ of complex numbers converges to z? When we say this sequence is a Cauchy Sequence? (E, CO 3)
2. Suppose $z, w \in l^2(Z)$ and $l \in N$. Then prove that $U^l(z * w) = U^l(z) * U^l(w)$. (An, CO 4)
3. Define summable sequence of complex numbers. Hence, define $l^1(Z)$. (A, CO 4)
4. If $\psi_{-j,k} = R_{2^j k} f_i$ and $\phi_{-j,k} = R_{2^j k} g_j$, prove that $\hat{\phi}_{-j,k}(m) = e^{-2\pi i 2^j km/N} \hat{\psi}_{-j,0}(m)$ and $\hat{\phi}_{-j,k}(m) = e^{-2\pi i 2^j km/N}\, \hat{\phi}_{-j,0}(m)$. (E, CO 2)
5. If $z = (1, 2, 0, 4)$, find $z(9), z(-2)$ and $z(0)$. (A, CO 1)
6. If $u_1, v_1, u_2, v_2, \ldots u_p, v_p$ is the p^{th} stage wavelet filter sequence, describe the output of the analysis phase of the p^{th} stage wavelet filter bank. (An, CO 2)
7. Prove that $z * w = w * z$ for any $z, w \in l^2(Z_N)$. (U, CO 1)
8. When we say a complex valued function f defined on $[-\pi, \pi)$ is square integrable over $[-\pi, \pi)$?. (An, CO 3)
9. Define the trigonometric system. Hence define a trigonometric polynomial. Is $sin(\theta - \theta_0)$ a trigonometric polynomial ? justify. (A, CO 3)
10. When we say a square matrix is unitary?. Prove that $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is unitary. (An, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. i) let $\{a(n)\}_{n\in Z}$ be a sequence of non-negative real numbers such that $|w(n)| \leq a(n)$ for all n such that $|n| \geq N$ for some N∈Z. If $\sum_{n\in Z} a(n)$ converges, prove that $\sum_{n\in Z} w(n)$ Converges. (A, CO 3)

    ii) If $\sum_{n\in Z} w(n)$ Converges absolutely, prove that $\sum_{n\in Z} w(n)$ Converges.
12. (a) If $z = (z(0), z(1), \ldots, z(N-1)) \in l^2(Z_N)$, what is $\overline{z}$? Prove that $(\overline{z})^{\wedge}(m) = \overline{\hat{z}(N-m)} : 0 \leq m \leq N-1$ (U, CO 1)
13. Let $\hat{u} = (\sqrt{2}, 1, 0, 1)$ and $\check{v} = (0, 1, \sqrt{2}, -1)$ (A, CO 1)

    (a) Find u and v

    (b) Construct an orthonormal basis for $l^2(Z_4)$ using u and v

14. Give an example of a basis for $l^2(Z_N)$ with justification which is (i) spatially localized but not frequency localized and (ii) frequency localized but not spatially localized. (A, CO 1)

15. (i) Let $w \in l^2(Z)$. Then prove that $\{R_{2k}w\}_{k\in Z}$ is orthonormal if and only if <w, R_{2k}w> = 1 if k = 0
<w, R_{2k}w> = 0 if k ≠ 0
for all k ∈ Z.
(ii) Suppose $z, w \in l^2(Z)$. Prove that
(a) $U(z * w) = U(z) * U(w)$.
(b) $[U(z)]^{\sim} = U(\tilde{z})$.
(c) $(z * w)^{\sim} = \tilde{z} * \tilde{w}$. (An, CO 4)

16. i) Prove that $e^{in\theta}$ is square integrable over $[-\pi, \pi)$ for all $n \in Z$.
ii) Prove that the trigonometric system is an orthonormal set in $L^2([-\pi, \pi))$. (U, CO 3)

17. Suppose N is divisible by 2^p.Suppose u,v $\in l^2(Z_N)$ are such that the system matrix A(n) of u and v is unitary for all n.
Define $u_1 = u$ and $v_1 = v$ and for $l = 2, 3, . . , p$ define
$u_l(n) = \sum_{k=0}^{2^{l-1}-1} u_1(n + \frac{kN}{2^{l-1}})$ and $v_l(n) = \sum_{k=0}^{2^{l-1}-1} v_1(n + \frac{kN}{2^{l-1}})$. Then prove that $u_1, v_1, u_2, v_2, \ldots u_p, v_p$ is a p^{th} stage wavelet filter sequence. (An, CO 2)

18. Suppose N is divisible by 2^p and let $u_1, v_1, u_2, v_2, \ldots u_p, v_p$ be a p^{th} stage wavelet filter sequence. With the usual notations prove that $f_1, f_1, \ldots., f_p, g_p$ generate a p^{th} stage wavelet basis for $l^2(Z_N)$. (E, CO 2)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. (a) Let $w \in l^2(Z_N)$. Then prove that $\{R_k w\}_{k=0}^{N-1}$ is orthonormal basis for $l^2(Z_N)$ if and only if $|\hat{w}(n)| = 1$ for all $n \in Z_N$
(b) If $B = \{R_k w\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(Z_N)$. Prove that $[z]_B = z * \tilde{w}$. (U, CO 1)

20. i) Suppose $f \in L^1([-\pi, \pi))$ and $< f, e^{in\theta} >= 0$ for all $n \in Z$. Then prove that $f(\theta)$=0 a.e
ii) Prove that the trigonometric system is complete in $L^2([-\pi, \pi))$. (R, CO 3)

21. With the usual notations prove that $V_{-l} \oplus W_{-l} = V_{-l+1}$. (An, CO 4)

22. Suppose N is divisible by $2^l, g_{l-1} \in l^2(Z_N)$ and the set $\{R_{2^{l-1}k} g_{l-1}\}_{k=0}^{\frac{N}{2^{l-1}-1}}$ is orthonormal and has $\frac{N}{2^{l-1}}$ elements. Suppose $u_l, v_l \in l^2(Z_{N/2}^{l-1})$ and the system matrix $A_l(n)$ is unitary for all $n = 0, 1, 2, \ldots, (N/2^l) - 1$. Define $f_l = g_{l-1} * U^{l-1}(v_l)$ and $g_l = g_{l-1}(*)U^{l-1}(u_l)$. With the usual notations prove that $V_{-l} \oplus W_{-l} = V_{-l+1}$ (An, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Define first stage and pth stage wavelet basis for l 2[ZN], Fourier transform including discrete case ,complete orthonormal system , first stage wavelet system and homogeneous wavelet system for l2[Z]	U	5, 7, 10, 12, 13, 14, 19	14
CO 2	Explain the filter bank diagram and its use in the construction of the output of the analysis phase of the filter bank	U	4, 6, 17, 18, 22	11
CO 3	Apply theory of wavelets in the frequency analysis of a video or audio signal.	U	1, 8, 9, 11, 16, 20	12
CO 4	Develop wavelet bases for l 2 [ZN] and l2 [Z],both first stage and pth stage	U	2, 3, 15, 21	9

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;