Reg. No .... Name **25U658**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT12 : FOURIER SERIES, LAPLACE TRANSFORMS AND METRIC SPACES**

(For Regular 2022 Admission and Supplementary 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the Fourier coefficienta_0 for the function $f(x) = xsinx$ in the interval -π<x<π
2. Give an example of a metric on $\mathbb{R}$ other than the usual metric.
3. Give an example of a sequence which is not convergent but has a limit point for its set of points.
4. Define Laplace transform and find the Laplace transform of 1
5. Explain periodic functions. Sketch the graph of the periodic function $f(x) = x$ with period 2π from -∞ to ∞
6. Find the set of all limit points of the sets $\mathbb{N}$ and $\mathbb{Q}$ in $X = \mathbb{R}$ with the usual metric.
7. What do you mean by a Cantor set?
8. Find the Laplace transform of $e^{at}\cos at$
9. Write the formula for half range Fourier cosine series of a function defined in the interval 0,l
10. Find the inverse Laplace transform of $\frac{3s+5\sqrt(2)}{s^2+8}$.
11. Give an example of a function which is continuous but not uniformly continuous on (a) $\mathbb{R}$ (b) $(0, 1)$.
12. Define a Cauchy sequence. Give example.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the inverse Laplace transform of $\frac{1-\cos t}{t}$
14. Prove that if $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then there exists a point in X which is not in any of the A_n's
15. Find the Fourier series of $f(x) = \sin x,\ 0 < x < 2\pi$
16. Let X be a non-empty set, and let $d : X \times X \to \mathbb{R}$ be a function which satisfies the following: $d(x, y) = 0 \Leftrightarrow x = y$ and $d(x, y) \leq d(x, z) + d(y, z)$. Show that d is a metric on X.
17. Find the Laplace transform of $t^2e^{-2t}\cos t$
18. Let X be a complete metric space and Y be a closed subspace of X, then prove that Y is complete.
19. Let X be a metric space and A be a subset of X. Prove that Int(A) is an open subset of A which contains every open subset of A.
20. Obtain cosine and sine series for $f(x) = x$ in the interval $0 \leq x \leq \pi$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. Prove that every non-empty set on the real line is the union of a countable disjoint class of open intervals.

22. State and prove the necessary and suffcient sequential criterion for a function $f : X \to Y$ to be continuous at a point $x_0 \in X$, where X and Y are metric spaces.

23. Find a series of cosines of multiples of x which will represent $xsinx$ in the interval $(0, \pi)$ and show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots = \frac{\pi - 2}{4}$.

24. a) Solve $y" + y' - 2y = t, y(0) = 1,\ y'(0) = 0$
    b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$

(10 x 3 = 30)