Reg. No .... Name **25P2067**

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2025

SEMESTER 2 : MATHEMATICS

COURSE : **24P2MATT10 : MEASURE THEORY AND INTEGRATION**

(For Regular 2024 Admission)

Time : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. A countable set has zero outer measure. Justify. (A)
2. Show that outer measure is translation invariant. (A)
3. Define Lebesgue integral of a simple function. (U)
4. State Tonelli's theorem (R)
5. Define upper and lower Riemann integrals using step functions. (A)
6. Define counting measure and Dirac measure. (U)
7. Define Lebesge measure. (R)
8. Define a positive and negative set. (R)
9. State Chebychev's inequality over general measure space. (U)
10. Does there exist a non measurable function? Justify. (A)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If E_1 and E_2 are measurable then show that $E_1 \cup E_2$ is measurable. (An)
12. Let (X, M, μ) be a measure space. Show that
    (i) if E_1 and E_2 are measurable and $\mu(E_1 \Delta E_2) = 0$, then $\mu(E_1) = \mu(E_2)$ (A)
    (ii) if μ is complete, $E_1 \in M$ and $E_2 \sim E_1 \in M$, then $E_2 \in M$ if $\mu(E_1 \Delta E_2) = 0$.
13. State and prove Simple approximation lemma. (An)
14. Let ϕ and ψ be simple functions defined on a set of finite measure E. Then for any α and β, show that
    (i) $\int_E (\alpha\phi + \beta\psi) = \alpha \int_E \phi + \beta \int_E \psi$. (An)
    (ii) If $\phi \leq \psi$ on E, then $\int_E \phi \leq \int_E \psi$.
15. Let (X, M, μ) be a measure space and f a measurable function on X. If f is bounded on X and vanishes outside a set of finite measure , then prove that f is integrable over X. (A)
16. Show that the class of all measurable sets is an algebra. (A)
17. Prove that the union of a finite collection of measurable sets is measurable with respect to μ^*.

(A)

18. Let (X, M, μ) be a measure space and f a nonnegative measurable function on X.Then prove that there is an increasing sequence $\{\psi_n\}$ of simple functions on X that converges pointwise on X to f and $lim_{n\to\infty} \int_X \psi_n d\mu = \int_X f d\mu.$ (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove Radon Nikodym theorem. (An)

20. a) State and prove Hahn's lemma.
b) State and prove Hahn's theorem. (A)

21. A) Let ϕ be the Cantor Lebesgue function and define the function ψ on $[0,1]$ by $\psi(x) = \phi(x) + x$ for all $x \in [0,1]$. Then show that ψ is a strictly increasing continuous function that $[0,1]$ onto $[0,2]$,
(i) maps the Cantor set onto a measurable set of positive measure and
(ii) maps a measurable set, a subset of the Cantor set, onto a nonmeasurable set (An)
B) Show that there is a measurable set, a subset of the Cantor set, that is not a Borel set.

22. State and prove any two convergence theorems. (An)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;