Reg. No ..... Name **25U461**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 4 : MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS/CHEMISTRY)

COURSE : **19U4CPMAT04 : Fourier Series,Laplace Transforms, Fourier Transforms, and Groups.**

(For Regular 2023 Admission and Improvement/Supplementary 2022/ 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. State the Dirchlet's conditions
2. Find the Laplace transform of $\cosh at$
3. Define Laplace transform and find the Laplace transform of e^{ax}
4. Define Laplace transform and find the Laplace transform of $\cos ax$
5. Write the inversion Formula of the Fourier Sine Transform.
6. Explain periodic functions. Sketch the graph of the periodic function $f(x) = x$ with period 2π from -∞ to ∞
7. Is the set of rational numbers $\mathbb{Q}$, under multiplication, a group? Justify your answer.
8. Give an example of an infinite group that is not cyclic.
9. Define the Complex Fourier Transform.
10. Write the inversion Formula of the complex Fourier Transform.
11. Explain a periodic function with example
12. How many elements does the symmetric group S_n have?

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the inverse Laplace transform of $\frac{2s^2-1}{(s^2+1)(s^2+4)}$
14. Show that the identity element in a group is unique.
15. Obtain Fourier series of the function $f(x) = \begin{cases} \pi x & ; 0 \le x \le 1 \\ \pi(2-x) & ; 1 \le x \le 2 \end{cases}$
16. Find the Fourier sine transform of $2e^{-5x} + 5e^{-2x}$.
17. Explain the group D_4. Write down its elements and its subgroups.
18. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$
19. Find the Fourier series of the function $f(x) = x^2, 0 < x < 2\pi$
20. Find the Fourier sine transform of $f(t) = \frac{1}{t}e^{-at}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (a) Let S be the set of all real numbers except -1. Define $*$ on S by $a * b = a + b + ab$. Show that $< S, * >$ is a group.
(b) Show that the left and right cancellation laws hold in any group $< G, * >$.
22. Find the Fourier transform of $f(x) = \begin{cases} e^x, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$.

23. Find the Fourier series to represent the function $f(x)$ given by $f\left(x\right) = \begin{cases} x & 0 \leq x \leq \pi \\ 2\pi - \mathrm{x} & \pi \leq \mathrm{x} \leq 2\pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

24. a) Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,\ x\left(0\right) = 0, x'\left(0\right) = 1$

b) Apply convolution theorem to find the inverse Laplace transform of $\frac{s^2}{s^4 - a^4}$

(10 x 3 = 30)