Reg. No .... Name **25U2004**

END SEMESTER EXAMINATION - APRIL 2025

SEMESTER 2 - INTEGRATED M.Sc. PROGRAMME COMPUTER SCIENCE - DATA SCIENCE

COURSE : **21UP2CPCMT02 - MATHEMATICS - II - LINEAR ALGEBRA**

(For Regular - 2024 Admission and Improvement / Supplementary - 2023/2022/2021 Admissions)

Time : Three Hours Max. Weightage : 30

PART A
Answer any 8 Questions

1. Define Linear Map with an Example.
2. Check whether the following vectors in F^3 can be written as a linear combination of the vectors (2,1,-3) and (1,-2,3);
    1. (17,-4,2)
    2. (17,-4,5)
3. Prove the following result;
    1. For each fixed u ∈V, the function that takes V to < v,u > is a linear map from V to F.
    2. < 0,u> = 0 for every u ∈ V.
4. Give matrix representarion for the following operators; T ∈ L(F^2) defined by T(x,y) = (2x+3y, 5x).
5. Define range of a transformation and give the range of the zero transformation.
6. Define Basis of a vector space and give the standard basis for F^2,P^2 and F^3
7. Prove that a0 = 0 for every a ∈ F.
8. Suppose v ∈ V ,then prove the following;
    1. ‖v‖ =0 if and only if v=0
    2. ‖ λ v‖ = |λ │ ‖v‖ for all λ ∈ F
9. Define orthonormal list of vectors.
10. Define Eigenspace and diagonalizable operator.

(1 x 8 = 8 Weight)

PART B
Answer any 6 Questions

11. Check whether T ∈ L(P^2) defined by T(at^2+bt+c) = (5a+b+2c)t^2 + 3bt +(2a+b+5c) is diagonalizable with respect to the basis t^2-2t,-2t+1,t^2+1 of P^2.Give valuable reason for your answer.
12. Suppose V and W are finte dimensional vector spaces such that dimV > dimW .Then prove that no linear map from V to W is injective
13. Check the following list of vectors are orthonormal list in F^3,
$\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}}\right)$
14. Suppose T ∈ L(V,W) .Then prove that nullT is a subspace of T
15. Suppose V is finite dimensional ,Then prove that every orthonormal list of vectors in V can be extended to an orthonormal basis of V

16. Suppose that U and W are subspaces of V.Then Prove that U+W is a direct sum if and only if $U \cap W = 0$

17. Check whether $T \in L(P^1)$ defined by T(at+b) = (4a+3b)t +(3a-4b) is diagonalizable with respect to the basis 3t+1,-t+3 of P^1.Give valuable reason for your answer.

18. Check whether the list (1,2,1), (2,1,0),(1,-2,2) is a basis in F^3

(2 x 6 = 12 Weight)

PART C
Answer any 2 Questions

19. Suppose $v_1,\ldots,v_n$ is basis of V and $w_1,\ldots,w_n \in W$. Then prove that there exists a unique linear map T: V→W such that $Tv_j = w_j$

20. Find $u \in P_2(R)$ such that $\int_{-1}^{1} p\left(t\right) \cos\left(\pi t\right) dt = \int_{-1}^{1} p\left(t\right) u\left(t\right) dt$ for every $p \in P_2(R)$

21. Check whether the following transformations are diagonalizable over the given basis,give valuable reason for your answers.

   1. $T \in L(P^1)$ defined by T(at+b) = (2a-3b)t +(a-2b) with respect to the basis 3t+1,t+1 of P^1
   2. $T \in L(P^1)$ defined by T(at+b) = (4a+3b)t +(3a-4b) with respect to the basis 3t+1,
   - t+3 of P^1
   3. $T \in L(P^2)$ defined by $T(at^2+bt+c) = (5a+b+2c)t^2 + 3bt + (2a+b+5c)$ with respect to the basis $t^2-2t,-2t+1,t^2+1$ of P^2

22. Prove that a list $v_1,\ldots,v_n$ of vectors of V is a basis of V if and only if every $v \in V$ can be written uniquely in the form
$v = a_1v_1+\ldots+a_nv_n$, $a_1,\ldots,a_n \in F$.

(5 x 2 = 10 Weight)