Reg. No .... Name **25P2019**

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2025

SEMESTER 2 : MATHEMATICS

COURSE : **24P2MATT07 : COMPLEX ANALYSIS**

(For Regular 2024 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Define limit superior and limit inferior with an example	(A, CO 4)
2.	State the second and third version of Cauchy's theorem	(R)
3.	Prove that $e^{-z} = \frac{1}{e^z}$	(A)
4.	Show that $\lim_{n\to\infty} n^{1/n} = 1$	(U)
5.	For the $f(z) = \frac{sinz}{z}$ has an isolated singularity at $z = 0$. Determine its nature; if it is a removable singularity define $f(0)$ so that f is analytic at $z = 0$; if it is a pole find the singular part	(An)
6.	Evaluate $\int_\gamma \frac{dz}{z-a}, \gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$	(A)
7.	Define removable singularity? Give an example of a function with a removable singularity and non removable singularity.	(U)
8.	Find all entire functions f such that $f(x) = e^x$ for $x \in \mathbb{R}$	(E)
9.	State the second version of the Maximum Modulus theorem and give the importance of boundedness in it	(A, CO 4)
10.	Define cross ratio? Evaluate the cross ratio $(7 + i, 1, 0, \infty)$	(A)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If $\gamma_1, \gamma_2 . . . . \gamma_m$ are closed rectifiable curves in G such that $n(\gamma_1; w) + n(\gamma_2; w) + ... n(\gamma_m; w) = 0$ for all w in $\mathbb{C} - G$ then show that for a in $G - \gamma$ and $k \geq 1$, $f^k(a) \sum_{j=1}^m n(\gamma_j, a) = k! \sum_{j=1}^m \frac{1}{2\pi i} \int_{\gamma_j} \frac{f(z)}{(z-a)^{k+1}} dz$	(A)
12.	If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G then prove that f has a primitive in G	(A)
13.	If $f : G \longrightarrow \mathbb{C}$ is analytic , then show that f preserves angles and each point z_0 of G where $f^{'}(z_0) \neq 0$	(An, CO 1)
14.	Let $f(z) = \frac{1}{z^2(4z-1)}$ find the Laurent series expansion valid in the region a) $0 \leq \lvert z\rvert \leq 1/4$ and b) $\lvert z\rvert \geq 1/4$	(A, CO 3)
15.	If $\lvert a\rvert \leq 1$ then prove that $\phi_a(z) = \frac{z-a}{1-\bar{a}z}$ is a one one map of $D = \{z : \lvert z\rvert \leq 1\}$ onto itself; the inverse of ϕ_a is ϕ_{-a}. Furthermore, ϕ_a maps ∂D onto ∂D, $\phi_a(a) = 0$, $\phi'_a(0) = 1 - \lvert a\rvert^2$ and $\phi'_a(a) = (1 - \lvert a\rvert^2)^{-1}$	(A, CO 4)
16.	State and prove Casorati- Weierstrass theorem	(R, CO 3)

17. Discuss the transformation $w = z^n$ and $w = z^{1/n}$ (A)

18. State and prove the Schwarz's lemma (R, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ have radius of convergence $R \geq 0$ then prove the following
a) For each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)\ldots\ldots\ldots\ldots(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R
b) The function f is infinitly differentiable on $B(a;R)$ and furthermore, $f^{(k)}(z)$ is given by the series $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)a_n(z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| \leq R$
c) For $n \geq 0$,$a_n = \frac{1}{n!}f^{(n)}(a)$ (U, CO 1)

20. Suppose f is analytic on D with $|f(z)| \leq 1$, also suppose $|a| < 1$ and $f(a) = \alpha$, hence $|f(z)| < 1, \forall z \in D$ and so f maps D into D. Among all functions f having these properties what is the maximum possible value of $|f'(a)|$? (A, CO 4)

21. Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{(2)}}$ (A, CO 3)

22. Let γ be a closed rectifiable curve in $\mathbb{C}$. Then prove that $n(\gamma; a)$ is a constant for a belonging to a component of $G = \mathbb{C} - \gamma$. Also $n(\gamma; a) = 0$ for a belonging to the unbounded component of G (R)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Understand concept of representation of complex numbers in the extended complex plane.	U	13, 19	7
CO 3	Represent analytic functions as power series	U	14, 16, 21	9
CO 4	Identify zeros and classify singularities of complex function	U	1, 9, 15, 18, 20	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;