B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025
SEMESTER 4 : MATHEMATICS FOR COMPUTER APPLICATION
COURSE : **19U4CRCMT5 : DIFFERENTIAL EQUATIONS**
(For Regular 2023 Admission and Improvement/Supplementary 2022/ 2021/2020/2019 Admissions)

Time : Three Hours   Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Find the general solution of $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 5y = 0$.

2. Verify that $z = f(x^2 + y^2)$ is a solution of $y\,\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$.

3. Locate and classify the singular points of the differential equation $x^2(x-2)^2\frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+1)y = 0$.

4. Solve $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$.

5. Find the integrating factor of the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$.

6. Solve $\tan\theta\,dr + 2r\,d\theta = 0$.

7. Find the integrating factor of $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

8. Find the ordinary points of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$.

9. Find the general solution of $\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$.

10. Find the general solution of $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$.

11. Show that the function $u = e^x\,\cos\,y$ is a solution of the two dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

12. Solve $p + q = x + y + z$.

                                                                                 (2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the general solution of $3x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 2y = 0$.

14. Solve the differential equation $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$.

15. Prove that $\frac{d}{dx}(x\,J_1(x)) = x\,J_0(x)$.

16. Find the general solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \cos\,4x$.

17. Show that a family of spheres $x^2 + y^2 + (z-c)^2 = a^2$ where a and c are arbitary constants, satisfies the first-order partial differential equation $yp - xq = 0$.

18. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2e^x$.

19. Solve $(z-y)p + (x-z)q = y - x$.

20. Find a family of oblique trajectories that intersect the family of curves $x + y = cx^2$ at angle α such that $\tan\,\alpha = 2$.

                                                                                 (5 x 5 = 25)

Answer any 3 (10 marks each)

21. Solve the Cauchy-Euler equation $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$.

22. Solve the initial value
    problem $(2x + 3y + 1)dx + (4x + 6y + 1)dy = 0.$ $y(-2) = 2.$

23. i) Solve the partial differential equation $z(xp - yq) = y^2 - x^2$.

    ii) Form the partial differential equation from the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by
    eliminating the arbitary constants.

24. Find the power series solution in powers of x of the differential
    equation $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$

(10 x 3 = 30)