Reg. No ..... Name **25U428**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 4 : MATHEMATICS

COURSE : **19U4CRMAT4 : ANALYTIC GEOMETRY NUMERICAL METHODS AND NUMBER THEORY**

(For Regular - 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that if p is a prime and $k > 0$, then $\phi(p^k) = p^k - p^{k-1}$.
2. Find the latus rectum, eccentricity and coordinates of foci of $4x^2 + 9y^2 = 36$.
3. Describe a computational procedure to implement NewtonRaphson method for computing the square root of a positive number.
4. Show that if $a \equiv b(\mod n)$ and $c \equiv d(\mod n)$, then $a + c \equiv b + d(\mod n)$.
5. Find the equation of the directrix of the conic $r \sin^2 \frac{\theta}{2} = 1$.
6. Find the equation of director circle to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
7. For what point of the parabola $y^2 = 18x$ is the ordinate equal to three times the abscissa?
8. Find the points on the conic $\frac{9}{r} = 2 + \sqrt{2}\cos\theta$ whose radius vector is 3.
9. Show that if p is a prime, then $a^p \equiv a(\mod p)$ for any integer a.
10. Find the nature of the conic $\frac{5}{r} = 2 - 2\cos\theta$.
11. Use bisection method to find the real root of the equation $f(x) = x^3 - x - 1 = 0$.
12. Find the equation of the parabola whose focus is $(5, 3)$ and directrix is $3x + 2y + 7 = 0$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Use Newton-Raphson method to find a root correct to three decimal places, of the equation $\sin x = x/2$, given that the root lies between $\pi/2$ and π.
14. Show that the locus of a point which moves such that the difference of its distances from two fixed points is a constant , is a hyperbola.
15. Use the method of iteration to find, correct to four significant figures, a real root of the equation $1 + x^2 = x^3$.
16. Find the asymptotes of $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$.
17. Prove that the polar of a point with respect to circle is perpendicular to the line joining the centre and the point.
18. Show that the locus of all points the sum of whose distances from two fixed points is constant is an ellipse.
19. If n is an odd pseudoprime, show that $M_n = 2^n - 1$ is a larger one.
20. If the integer $n > 1$ has the prime factorization $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$, then prove that $\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_r}\right)$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Prove that ϕ is a multiplicative function.
22. Find the equation of the chord joining $\theta = \alpha - \beta$ and $\theta = \alpha + \beta$.
23. If θ and ϕ be the eccentric angles of the ends of a focal chord of an ellipse of eccentricity e, prove that
    $\pm e \ \cos\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$.
24. Using NewtonRaphson method, derive a formula for finding the k^{th} root of a positive number N and hence compute the value of $25^{1/4}$.

(10 x 3 = 30)