Reg. No .... Name **25P2003**

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2025

SEMESTER 2 : MATHEMATICS

COURSE : **24P2MATT06 : BASIC TOPOLOGY**

(For Regular - 2024 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Show that any function into a trivial space is continuous.	(R)
2.	Let X be an infinite set where the closed sets are finite sets. Is X Hausdorff ?	(U)
3.	Prove that compact subsets in a Hausdorff space are closed.	(U)
4.	Define a lebesgue number.	(R)
5.	Let X be a connected space. Show that only clopen subsets of X are X and ϕ.	(A)
6.	Define closed set and open sets. Give examples of sets that are (i) both open and closed (ii) neither open nor closed.	(R)
7.	Let $Z \subset Y \subset X$ and $\mathscr{T}$ be a topology on X then, prove that $\mathscr{T}/\mathrm{Y}/Z = \mathscr{T}/Z$.	(U)
8.	Show that the inclusion function is continuous.	(U)
9.	Define closure operator on a topological space $(X, \mathscr{T})$.	(R)
10.	Define (i) Path connected (ii) Locally connected.	(U)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	A space is locally connected at a point x if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.	(R)
12.	Show that every path connected space is connected. Is the converse true ? Justify your answer.	(U)
13.	Suppose (X, τ) be a space and $Y \in \tau$. Prove that a subset B of Y is open in Y if and only if it is open in X.	(R)
14.	Prove that every T_3 space is T_2 but the converse is not true.	(U)
15.	Prove that normality is a weakly hereditary property.	(U)
16.	If a space is second countable then prove that every open cover of it has a countable subcover.	(R)
17.	Show that closed subspace of compact space is compact.	(U)
18.	Let X be a non empty set and $\mathscr{T} = \{G \subset X : X - G$ is countable$\} \cup \{\phi\}$. Prove that $\mathscr{T}$ is a topology on X.	(U)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19.	Define T_4 and prove that all metric spaces are T_4.	(U)

20. For a topological space X show that the following are equivalent:

(a) X is locally connected.

(b) Components of open subsets of X are open in X.

(c) X has a base consisting of connected subsets.

(d) For every $x \in X$ and every neighbourhood N of x there exists a connected open neighbourhood M of x such that $M \subset N$. (U)

21. Establish three equivalent conditions for a topological space $\mathscr{T}$ to have a base $\mathscr{B}$. (U)

22. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.(b) Prove that continuous image of a compact space is compact. (U)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;