Reg. No .... Name **25P4029**

MSc DEGREE END SEMESTER EXAMINATION- MARCH 2025

SEMESTER 4 : MATHEMATICS

COURSE : 21P4MATTEL18 : PROBABILITY THEORY

(For Regular - 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Define almost sure convergence. (R, CO 4)
2. A slip of paper is given to a person A, who marks it with either a plus or minus sign; the probability of her writing a plus sign is $\frac{1}{3}$. A passes the slip to B, who may either leave it alone or change the sign before passing it to C ; Next, C passes it to D after perhaps changing the sign; finally, D passes it to the referee after perhaps changing the sign. The referee sees a plus sign on the slip. It is known that B,C and D each change the sign with probability $\frac{2}{3}$.Find the probability that A originally wrote a plus. (E, CO 1)
3. Let A, B be two events such that $B \supseteq A$. What is $P(A \cup B), P(A \cap B), P(A - B)$? (A, CO 1)
4. Does $F(x) = 1 - e^{-x}, x \geq 0$ and $= 0, x < 0$ define a DF? (A, CO 2)
5. Let F_n be a sequence of DF's with $F_n(x) = 0, x < n; 1, x \geq n$.Does $F_n(x)$ converge to a DF? (U, CO 4)
6. Let X be the rv defined on a probability space (Ω, S, P) by $X(\omega) = c, \forall \omega \in \Omega$. Find the DF of X. (U, CO 2)
7. Let X be a rv with standard normal pdf , $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}, -\infty < x < \infty$. Find the truncated pdf of X if $T = (-\infty, 0]$. (A, CO 3)
8. Define Distribution function of a multiple random variable. (R, CO 3)
9. Let (X, Y) be jointly distributed with pdf $f(x, y) = 2, 0 < x < y < 1$, and $= 0, otherwise$.Find the marginal pdf's. (A, CO 3)
10. Define moments of a random variable. (R, CO 2)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Let $X_n \sim U(0, \frac{1}{n})$. Show that $X_n \xrightarrow{r} 0$ for any $r \geq 1$. (A, CO 4)
12. Let A, B be two independent events defined on some probability space, and let $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$. Find $P(A \cup B), P(A|A \cup B)$ and$P(B|A \cup B)$. (A, CO 1)
13. State and prove total probability rule. (A, CO 1)
14. Show that two rv's X and Y are independent iff for every pair of Borel measurable functions g_1 and g_2 the relation $E(g_1(X)g_2(Y)) = E(g_1(X))E(g_2(Y))$ holds, provided that the expectations on both sides of the equation exists. (An, CO 3)
15. Let X be a rv on a probability space (Ω, S, P).Let $E|X|^k < \infty$ for some $k > 0$. Then prove that $n^k P(|X| > n) \to 0$ as $n \to \infty$. (A, CO 2)

16. Let X have the density $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, Otherwise \end{cases}$. Find the pdf of $Y = e^X$ and $Y = -2logX$. (A, CO 2)

17. State Khinchin's theorem.Examine if WLLN holds for the $\{X_n\}$ of iid rv's with $P(X_i = (-1)^{k-1}k) = \frac{6}{\pi^2 k^2}, k = 1, 2, 3, \ldots, i = 1, 2, 3, \ldots$. (A, CO 4)

18. Suppose that X, Y, Z have joint pdf $f(x, y, z) = \begin{cases} \frac{2}{3}(x + y + z), 0 < x < 1, 0 < y < 1, 0 < z < 1 \\ 0, otherwise \end{cases}$.Then show that X, Y, Z are exchangeable, but not independent. (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Show that convergence in probability implies convergence in distribution.When does convergence in distribution imply convergence in probability ? Justify? (An, CO 4)

20. State Bayes theoem.A diagonostic test for a certain disease is 95 percent accurate, in that if a person has the disease, it will detect it with probability of 0.95, and if the person does not have the disease , it will give a negative result with probability of 0.95. Suppose that only 0.5 percent of the population has the disease .A person is chosen at random from this population. The test indicates that this person has the disease. What is the conditional probability that he or she does have the disease ? (E, CO 1)

21. Define conditional expectation for discrete and continuous rv's. If (X, Y) has joint pmf $p(0, 0) = \frac{4}{25}, p(0, 1) = \frac{6}{25}, p(1, 0) = \frac{6}{25}, p(1, 1) = \frac{9}{25}$ then find $E(X|Y)$ and $E(Y|X)$. (A, CO 3)

22. Let X be the rv defined on a probability space (Ω, S, P). Define F on R by $F(x) = Q(-\infty, x] = P(\omega : X(\omega \leq x))$ for all $x \in R$. Show that F is a distribution function of rv X. (An, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	define the principal concepts about probability and evaluating it.	A	2, 3, 12, 13, 20	11
CO 2	explain the concept of a random variable and the probability distributions	A	4, 6, 10, 15, 16, 22	12
CO 3	Analyze the concept of function of rv's and multiple rv's	An	7, 8, 9, 14, 18, 21	12
CO 4	Analyze the concept of convergence of sequence of rv's	An	1, 5, 11, 17, 19	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;