Name **25U640**

Reg. No ....

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT11 - LINEAR ALGEBRA AND GRAPH THEORY**

(For Regular 2022 Admission and Supplementary 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Draw the graph corresponding to the incIdence matrix

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

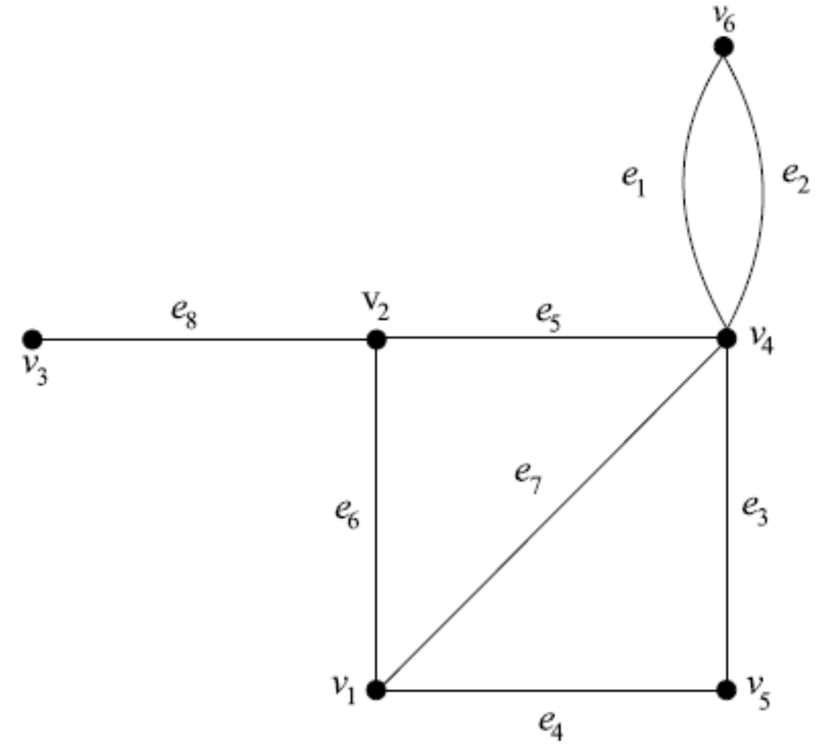
2. Find f (2), f (5), and f (-5) for f (x) = $1/x^2$
3. Explain the travelling salesman's problem
4. Explain the Chinese postman problem.
5. Determine which of the following sets are spanning sets for $\mathbf{R}^2$**,** considered as column matrices:

$$\text{(a)} \quad \mathbb{S}_1 = \left\{ \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$\text{(b)} \quad \mathbb{S}_2 = \left\{ \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad f_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$\text{(c)} \quad \mathbb{S}_3 = \left\{ \boldsymbol{f}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{f}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

6. Define M alternating path in a graph with an example.
7. Let **T:** $\mathbf{R}^2 \rightarrow \mathbf{R}^2$ is defined by T [a b] = [a+2 b-2] . Find
   a) T [2 3] b) T [-1 5]
   c) T [-8 200] d) T [0 -7]
8. If a graph G has 15 edges and all vertices of the same degree d, what are the possible values of d? Describe briefly each graph.
9. Determine whether the transformation **T: V → W** defined by T(v) = 0 for all vectors v in **V** is linear.
10. Define linearly dependent and linearly independent set of vectors in a vectorspace.
11. Define dimension of a vector space
12. Find the incidence matrix of the graph



(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

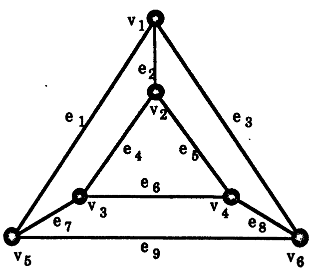
13. Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices

14. Prove that Every basis for a finite-dimensional vector space must contain the same number of vectors.

15. a) Draw K_5 and mark a maximum matching in the graph
b) Draw all trees with 6 vertices
c) Is K_n Eulerian

16. Determine whether the set Mp×n of all p×n real matrices under matrix addition and scalar multiplication is a vector space.

17. Write down the adjacency matrix and the incidence matrix for the graph



(b)

18. A linear transformation **T: R^2 → R^3** has the property that

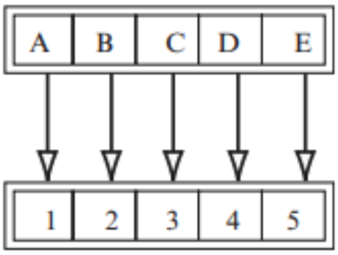
$$\boldsymbol{T}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\2\\0\end{bmatrix} \quad \text{and} \quad \boldsymbol{T}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\3\\4\end{bmatrix}$$

Determine T(v) for any vector v in **R^2**.

19. Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

20. The rules of correspondence are described by arrows. Determine whether the given relationships are functions and, for those that are, identify their image.

(1)



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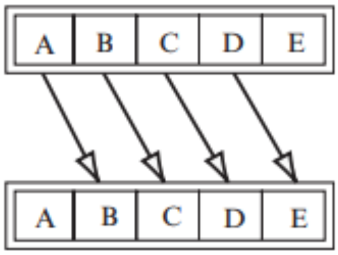

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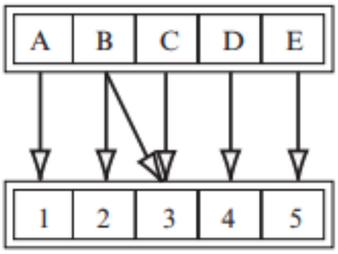

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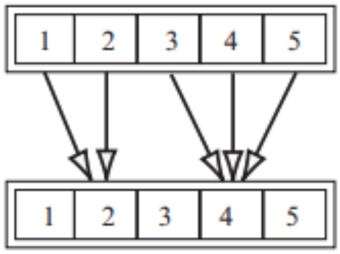
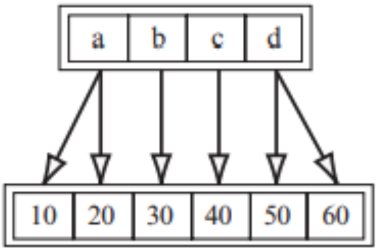


(5)



(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Use row rank to determine whether the following sets are linearly independent or not.

a) $\{[1 \quad 1 \quad 0], [1 \quad -1 \quad 0]\}.$

b) $\{[1 \quad 2 \quad 3], [-3 \quad -6 \quad -9]\}.$

c) $\{[10 \quad 20 \quad 20],\ [10 \quad -10 \quad 10],\ [10 \quad 20 \quad 10]\}.$

22. State and prove Berge theorem

23. Let e be an edge of a graph G then prove that $\omega(G) \leq \omega(G-e) \leq \omega(G)+1$

24. Find matrix representations for the linear transformation **T : R^2 → R^2** defined by

$$\boldsymbol{T}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11a + 3b \\ -5a - 5b \end{bmatrix}$$

(a) with respect to the standard basis C = { [1 0]^T , [0 1]^T }

(b) with respect to the basis D = { [3 -1]^T , [1 -5]^T }

Then verify the equation

$$A_C^C = (P_C^D)^{-1} A_D^D P_C^D$$

(10 x 3 = 30)