Name **25U622**

Reg. No ....

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT10 : COMPLEX ANALYSIS**

(For Regular 2022 Admission and Supplementary 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define simple arc.
2. Explain the convergence of an improper integral.
3. Write the coefficient of z^{2n} in the Taylor series expansion of $\sin z$.
4. Classify the singularities of $\frac{1}{e^{1/z}}$.
5. Discuss the convergence of the series $\sum \cos(i/n)$.
6. Obtain the Taylor series of e^z about $z = 1$.
7. State Cauchy-Goursat theorem.
8. Discuss the existence of $\lim\limits_{z \to 0} \frac{z}{\bar{z}}$.
9. State Jordan cuve theorem.
10. Use definition to evaluate $f'(0)$, where $f(z) = \bar{z}$.
11. Give an example of function which has a non isolated singularity.
12. Use definition to evaluate $f'(0)$, where $f(z) = |z|^2$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Suppose that $\lim\limits_{z \to z_0} f(z) = w_0$ and $\lim\limits_{z \to z_0} F(z) = W_0$. Show that $\lim\limits_{z \to z_0} [f(z) - F(z)] = w_0 - W_0$.
14. Classify all the singularities of $f(z) = \frac{1}{z(e^z - 1)}$.
15. True or false: Let $z_n = r_n e^{i\theta_n}$ and $z = re^{i\theta}$. Then $z_n \to z$ if and only if $r_n \to r$ and $\theta_n \to \theta$ as $n \to \infty$. Justify.
16. State and prove Cauchy's inequality.
17. Evaluate $\int\limits_C \bar{z} \; dz$ when $C : z = e^{i\theta} \quad (-\pi/2 \le \theta \le \pi/2)$.
18. Evaluate $\sin^{-1} i$.
19. Prove that $\sum\limits_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.
20. Compute the residue of $f(z) = \dfrac{1}{z^2(1+z)}$ at $z = 0$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Evaluate $\int_{|z|=2} \frac{5z-2}{z(z-1)} dz$.

22. Let $f(z) = u + iv$ be analytic on a domain D. Write Cauchy Riemann equations and use the same to show that f is constant on D, if $2u + 3v = 4$ on D.

23. Show that the absolute convergence of a series of complex numbers implies the convergence of that series. Is the converse true? Justify.

24. Let C denote a positively oriented simple closed contour. If a function f is analytic inside and on C, then prove that $f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(s)ds}{(s-z)^{n+1}}$.

(10 x 3 = 30)