Reg. No .... Name **25U604**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2025

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT09 : REAL ANALYSIS - 2**

(For Regular 2022 Admission and Supplementary 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that a function which is uniformly continuous on an interval is continuous on that interval.
2. Show that the series $\sum \frac{(-1)^n}{n}|x|^n$, converges uniformly in $[-1,1]$.
3. Discuss the kind of discontinuity, if any, of the function $f(x) = \begin{cases} \frac{x-|x|}{x} & when\ x \neq 0 \\ 2 & when\ x = 0 \end{cases}$
4. Discuss the differentiability of the function $f(x) = \begin{cases} 2, & x \leq 1 \\ x, & x > 1 \end{cases}$ at $x = 1$
5. State and prove the symmetrical property of the Beta function.
6. Prove that for any two partitions P_1 and P_2 of $[a,b]$ and for a bounded function f, $L(P_1, f) \leq U(P_2, f)$.
7. Define the beta function $B(m,n)$.
8. State Cauchy's criterion for uniform convergence.
9. When is a partition P^* of $[a,b]$ said to be finer than another partition P of $[a,b]$?
10. State Weierstrass M-test for uniform convergence.
11. Define the improper integral $\int_a^{\infty} f\,dx$, where $x \geq a$.
12. Give an example of a function which is not Riemann integrable on $\mathbb{R}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Show that $\sum \frac{\cos n\theta}{n^p}$ is uniformly and absolutely convergent for all real values of θ, where $p > 1$.
14. Discuss the differentiability of the function $f(x) = \begin{cases} 2x-3, & 0 \leq x \leq 2 \\ x^2-3, & 2 < x \leq 4 \end{cases}$ at $x = 2$ and $x = 4$.
15. Prove that a function which is differentiable at a point is necessarily continuous at that point.
16. If n is a positive integer and $m > -1$, prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$.
17. If f and g are bounded and integrable functions on $[a,b]$, such that $f \geq g$, prove that $\int_a^b f\,dx \geq \int_a^b g\,dx$.
18. Evaluate $\int_0^a y^4 \sqrt{(a^2-y^2)}dy$.

19. Let $f(x) = \begin{cases} \frac{1}{2^n}, & when\, \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}, (n = 0, 1, 2, \ldots) \\ 0 & when\, x = 0 \end{cases}$.

Show that $\int_0^1 f\,dx = \frac{2}{3}$.

20. Show that the series $\sum(-1)^n \frac{x^2+n}{n^2}$, converges uniformly in every bounded interval, but does not converge absolutely for any value of x.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

22. Show that if a function f is continuous on a closed interval $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one point $\alpha \in (a, b)$ such that $f(\alpha) = 0$.

23. State and prove Darboux's theorem.

24. Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$ is uniformly convergent on $[0, k]$, where $k < 1$ and is pointwise convergent on $[0, 1]$.

(10 x 3 = 30)