Reg. No………………………………………….   Name………………………………………………

BA/BSc/BCOM DEGREE END SEMESTER EXAMINATION - NOVEMBER 2024

UGP (HONS.) SEMESTER - 1: DISCIPLINE SPECIFIC COURSE (MATHEMATICS)

COURSE: 24UMATDSC101 - GROUND ROOTS OF MATHEMATICS

(For Regular 2024 Admission)

Time : 2 Hours   Max. Marks : 70

PART A

(Each question carries 2 marks. A maximum of 10 marks can be scored from this part)

1. Find the negation of the proposition "Michael's PC runs Linux" and express this in simple English.
2. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.
3. Define homogeneous system and nonhomogeneous system.
4. Define augmented matrix.
5. Use implicit differentiation to find $\frac{dy}{dx}$ if $5y^2 + siny = x^2$.
6. Find $\frac{dy}{dx}$ if $y = 3x^8 - 2x^5 + 6x + 1$.
7. Find the interval on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.
8. Define critical point and stationary point.

PART B

(Each question carries 5 marks. A maximum of 30 marks can be scored from this part)

9. What is the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is running"?
10. What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"?
11. Using Gauss Elimination method, solve the linear system
$$3.0x_1 + 2.0x_2 + 2.0x_3 - 5.0x_4 = 8.0,$$
$$0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7,$$
$$1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1.$$
12. Using Gauss Elimination method, solve the linear system
$$3x_1 + 2x_2 + x_3 = 3,$$
$$2x_1 + x_2 + x_3 = 0,$$
$$6x_1 + 2x_2 + 4x_3 = 6.$$
13. Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $4x^2 - 2y^2 = 9$.
14. Find $f''(\frac{\pi}{4})$ if $f(x) = secx$.

15. Find all critical points of the functions $f(x) = x^3 - 3x + 1$ and $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$.

16. Find $\lim\limits_{x \to \frac{\pi}{4}}(1 - tanx)\sec(2x)$.

PART C

(Each question carries 15 marks. A maximum of 30 marks can be scored from this part)

17. (a) Show that $\neg(p \to q)$ and $(p \wedge \neg q)$ are logically equivalent.

    (b) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalence.

18. Using Gauss Elimination method, solve the linear system.

$$x_1 - x_2 + x_3 = 0,$$
$$-x_1 + x_2 - x_3 = 0,$$
$$10x_2 + 25x_3 = 90,$$
$$20x_1 + 10x_2 = 80.$$

19. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$.

    (b) Find an equation for the tangent line to the Folium of Descartes at the point (3/2, 3/2).

    (c) At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?

20. Define the second derivative test and find the relative extrema of the function $f(x) = 3x^5 - 5x^3$.