Reg. No .... Name **24P1035**

M.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2024

SEMESTER 1 : MATHEMATICS

COURSE : **24P1MATT03 : REAL ANALYSIS**

(For Regular 2024 Admission and Improvement/Supplementary 2023/2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Prove that $\lim_{n\to\infty}(1+\frac{1}{n})^n = e.$ (A)
2. If $f_1, f_2 \in \mathscr{R}(\alpha)$ on [a,b], then prove that $f_1 + f_2 \in \mathscr{R}(\alpha)$ on $[a,b]$. (U)
3. What is the Riemann-Stieltjes integral, and how does it differ from the Riemann integral? (U)
4. Prove or disprove: Every monotone function is a function of bounded variation on $[a,b]$. (A)
5. State Stone Weierstrass theorem. (An)
6. Show that $\lim_{x\to 0}\frac{b^x-1}{x} = b$ (A)
7. Show that a convergent series of continuous functions may have a discontinuous sum. (An)
8. Show that there exists an everywhere discontinuous limit function which is not Riemann integrable. (A)
9. Define a partition and when is a function f said to be of bounded variation. (A)
10. Prove that the set of discontinuities of a monotone function is countable. (An)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E, prove that f is continuous on E. (R)
12. Determine the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2n!}{(n!)^2}z^n$ (A)
13. Show that the function f defined on $[0,1]$ by $f(x) = \begin{cases} 1 & \text{;if } x \text{ is rational} \\ 0 & \text{;if } x \text{ is irrational} \end{cases}$ is not integrable on $[0,1]$ (A)
14. State and prove Cauchy criterion for uniform convergence of a series of functions. (R)
15. If f is continous on $[0,1]$ and if $\int_0^1 x^n f(x)dx = 0$ for $n = 0,1,2,\ldots$, then show that $f(x) = 0$ on $[0,1]$. (A)
16. Prove that f is Riemann Stieltjes integrable on [a,b] iff for $\epsilon > 0$, there exist partition P of [a,b] such that $U(P,f,\alpha) - L(P,f,\alpha) < \epsilon$. (R)

17. Let f be a function of bounded variation on [a,b]. Let the variation function $V(x)$ defined by $V(x) = \begin{cases} v_f(a,x) & \text{if } x \in (a,b] \\ 0 & \text{if } x = a \end{cases}$ (A)

Then prove that :

1) V is increasing function on [a,b]

2) $V - f$ is an increasing function on [a,b}

18. Is the function $f(x) = \sqrt{x}\, sin(1/x)$ if x≠0, $f(0) = 0$ is of bounded variation on [0,1] (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove the additive property of total variation. (An, CO 1)

20. Let f∈R on [a,b]. For a≤x≤b, put $F(x) = \int_a^x f(t)dt$. Then prove that F is continuous on [a,b]; furthermore, if f is continuous at a point x_0 of [a,b], then prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$ (R)

21. Suppose $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and define $f(x) = \sum_{n=0}^{\infty} c_n x^n \quad (|x| < R)$, then prove that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\epsilon, R-\epsilon]$, no matter which $\epsilon > 0$ is chosen and the function f is contiuous and differentiable in $(-R, R)$ with $f'(x) = \sum_{n=0}^{\infty} n c_n x^{n-1}$. (U)

22. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$ For what values of x does the series converge absolutely? On what intervals does it converge uniformly ? On what intervals does it fail to converge uniformly? Is f continuous wherever the series converges? Is f bounded? (A)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain the functions of bounded variations, rectifiable curves, paths and equivalence of paths	U	19	5

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;