Reg. No……………………………………………. Name…………………………………………… **24P1045**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2024

SEMESTER 1 : Computer Science (Artificial Intelligence)

COURSE: **24P1CAIT04 : MATHEMATICS FOR COMPUTATIONAL INTELLIGENCE**

(*For Regular 2024 Admission*)

Time: Three Hours Max. Weightage : 30

PART-A **Weight : 1**

Answer any 8 Questions

1. Consider the matrices

   $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}$

   Prove that $(A+B)^2 \neq A^2 + 2AB + B^2$.
2. Prove that $(\mathbb{R}\backslash\{0\}, \ \cdot)$ is an Abelian group.
3. Define Manhattan norm and Euclidean norm on $\mathbb{R}^n$.
4. Define distance and metric.
5. Compute the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ using Laplace expansion.
6. Write any 4 properties of determinant and trace of a square matrix.
7. Define:
   i. Difference Quotient
   ii. Derivative
   iii. Taylor polynomial
   iv. Taylor series
8. Compute the Taylor polynomial T_6 of $f(x) = x^4$ at $x_0 = 1$.
9. Define convex and concave function with an example.
10. Consider whether the following statements are true or false:
    i. The product of any two convex functions is convex.
    ii. The maximum of any two convex functions is convex.

(1 x 8 = 8)

PART- B **Weights : 2**

Answer any 6 Questions

11. Compute the following matrix products

    i. $\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$

ii. $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$

12. Determine whether the matrix given below is invertible and if so, then find the inverse.

$$A = \begin{bmatrix}1 & 2 & 1\\4 & 4 & 5\\6 & 7 & 7\end{bmatrix}$$

13. Consider $\mathbb{R}^2$ with $\langle\cdot,\ \cdot\rangle$ defined for all *x* and *y* in $\mathbb{R}^2$ as

$$\langle x, y\rangle = x^T\begin{bmatrix}2 & 0\\1 & 2\end{bmatrix}y$$

Is $\langle\cdot,\ \cdot\rangle$ an inner product?

14. Compute the distance between

$$x = \begin{bmatrix}1\\2\\3\end{bmatrix},\quad y = \begin{bmatrix}-1\\-1\\0\end{bmatrix}$$

using

i. $\langle x, y\rangle = x^Ty$

ii. $\langle x, y\rangle = x^TAy, \qquad A = \begin{bmatrix}2 & 1 & 0\\1 & 3 & -1\\0 & -1 & 2\end{bmatrix}$

15. Find the eigenvalues and eigenvectors of the 2×2 matrix $A = \begin{bmatrix}4 & 2\\1 & 3\end{bmatrix}$.

16. Consider $f(x_1, x_2) = {x_1}^2 + 2x_2$, where $x_1 = \sin t$ and $x_2 = \cos t$, then find $\frac{df}{dt}$.

17. i. Define partial derivative.

ii. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y) = (x + 2y^3)^2$.

18. Consider the function $f(x) = \frac{1}{2}x^TAx + b^Tx + c$, where A is strictly positive definite, which means that it is invertible. Derive the convex conjugate of $f(x)$.

Hint: Take the gradient of an appropriate function and set the gradient to zero.

(2 x 6 = 12)

PART- C

Answer any 2 Questions **Weights : 5**

19. Using Gaussian elimination, find all solutions of the inhomogeneous equation system $Ax = b$ with

$$A = \begin{bmatrix}0 & 1 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 1 & 1 & 0\\0 & 1 & 0 & 0 & 0 & 1\end{bmatrix},\quad b = \begin{bmatrix}2\\-1\\1\end{bmatrix}$$

20. Consider $\mathbb{R}^3$ with inner product

$$\langle x, y \rangle = x^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} y.$$

Furthermore, we define e_1, e_2, e_3 as the standard/canonical basis in $\mathbb{R}^3$.

Determine the orthogonal projection $\pi_U(e_2)$ of e_2 onto $U = span[e_1, e_3]$.

Hint: Orthogonality is defined through the inner product.

21. Compute all eigenspaces of

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}.$$

22. Compute the derivative $\frac{df}{dx}$ of the following function using the chain rule.

$f(z) = \sin(z)\,, \quad z = Ax + b, \quad A \in \mathbb{R}^{E \times D}, \; x \in \mathbb{R}^D, \quad b \in \mathbb{R}^E$

Where $\sin(\cdot)$ is applied to every element of z.

(5 x 2 = 10)