

**M.Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2024****SEMESTER 1 : MATHEMATICS****COURSE : 24P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS***(For Regular 2024 Admission and Improvement/Supplementary 2023/2022/2021 Admissions)*

Duration : Three Hours

Max. Weights: 30

**PART A****Answer any 8 questions****Weight: 1**

1. Prove that if  $n = 0$ ,  $\int_{-1}^1 P_n(x)dx = 2$ . (U)
2. Show that the zeros of the functions  $\sin x$  and  $\cos x$  are distinct and occur alternately. (U, CO 1)
3. Define a stable critical point. (R)
4. Express  $2 - 3x + 4x^2$  in terms of Legendre polynomials. (A)
5. Define critical point and isolated critical point for an autonomous system. (R)
6. State Liapunov's condition for stability. (U)
7. Give an example to show that there exists functions which do not satisfy the Lipschitz condition. (A)
8. Examine the existence and uniqueness of solution for the initial value problem  $y' = 1 + y^2$ ,  $y(0) = 0$ . (A)
9. Find  $P_n(1)$ . (U)
10. Explain the method of separation of variables. (U, CO 1)

**(1 x 8 = 8)****PART B****Answer any 6 questions****Weights: 2**

11. State and prove Sturm comparison theorem. (A, CO 1)
12. Solve the following initial value problem upto third approximation.  
 $dy/dx = x^2 - y$ ,  $y(0) = 0$  (A)
13. Find the general solution of the system  $dx/dt = x + y$ ,  $dy/dt = y$ . (A)
14. Show that  $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{3\sin x}{x^2} - 3\frac{\cos x}{x} - \sin x \right)$ . (A)
15. Solve  $a^2 u_{xx} = u_{tt}$  with boundary conditions  $u(0, t) = 0$ ,  $u(\pi, t) = 0$  and initial conditions  $u(x, 0) = f(x)$ ,  $u_t(x, 0) = g(x)$ . (An, CO 1)
16. Prove that  $(J_{\frac{1}{2}}(x))^2 + (J_{-\frac{1}{2}}(x))^2 = \frac{2}{\pi x}$ . (An)
17. Solve the system  $dx/dt = -3x + 4y$ ,  $dy/dt = -2x + 3y$ . (A)
18. Apply Picard's method to find the first two approximations to the initial value problem  
 $y'(x) = x + z$ ,  $z'(x) = x - y^2$ ,  $y(0) = 2$ ,  $z(0) = 1$ . (A)

**(2 x 6 = 12)****PART C****Answer any 2 questions****Weights: 5**

19. For the following linear system, find the general solution, differential equation of the paths and its solution. Sketch a few paths showing the direction of increasing  $t$  and discuss the stability of the critical point  $(0,0)$ .  
 $dx/dt = x$   $dy/dt = -y$  (A)

20. Derive Bessel's Integral formula. (E, CO 1)
21. Explain the orthogonality property of Sturm Liouville problem. (An, CO 1)
22. State and prove the existence and uniqueness theorem where  $f$  satisfies Lipschitz condition. (An)
- (5 x 2 = 10)**

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Summarize the concepts of Sturm Separation theorem and Sturm Liouville problems	A	2, 10, 11, 15, 20, 21	16

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;