Reg. No .... Name **24P1003**

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2024

SEMESTER 1 : MATHEMATICS

COURSE : 24P1MATT01 : LINEAR ALGEBRA

(For Regular - 2024 Admission and Improvement/Supplementary 2023/2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Verify whether $D(A) = 1$ is $3-$linear. (A, CO 3)
2. Let V, W and Z be vector spaces over the field F. Let T be a linear transformation from V to W and let U be a linear transformation from W to Z. Show that the composed function UT defined by $UT(\alpha) = U(T(\alpha))$ is a linear transformation from V into Z. (U)
3. Define basis of a vector space. Write a basis for $\mathbb{R}^2$ other than the standard ordered basis. (U, CO 1)
4. Let V be a n-dimensional vector space over F.What is the characteristic polynomial of the identity operator on V? What is the characteristic polynomial for the zero operator? (U)
5. Define a determinant function. (U, CO 3)
6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1, x_2) = (\sin x_1, x_2)$. Is T linear? (A)
7. Let V be a vector space over the field F. Show that the intersection of any collection of subspaces of V is a subspace of V. (A, CO 1)
8. Define hyperspace in a vector space. Give an example. (A)
9. Prove that the determinant of a triangular matrix is the product of its diagonal entries. (A, CO 3)
10. If $T^2 = T$, show that T is diagonalizable. (A)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let V_e be the subset of even functions, $f(-x) = f(x)$; let V_o be the subset of odd functions $f(-x) = -f(x)$. (An, CO 1)
    (a) Prove that V_e and V_o are subspaces of V.
    (b) Prove that $V_e + V_o = V$
    (c) Prove that $V_e \cap V_o = \{0\}$.
12. Let $\mathscr{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the basis for $\mathbb{C}^3$ defined by $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (2, 2, 0)$. Find the dual basis of $\mathscr{B}$. (A)
13. Verify whether $D(A) = (A_{11})^2 + 3A_{11}A_{22}$ is $3-$linear. (A, CO 3)
14. Let V be a finite dimensional vector space over a field F and T be a linear operator on V. Prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F. (A)
15. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W.Suppose that V is finite dimensional. Prove that rank(T)+nullity(T)= dim V. (A)

16. Let V be the real vector space of all polynomial function from $\mathbb{R}$ into $\mathbb{R}$ of degree 2 or less. Let t be a fixed real number and define $g_1(x)=1, g_2(x)=x+t, g_3(x)=(x+t)^2$. Prove that $\mathscr{B}=\{g_1,g_2,g_3\}$ is a basis for V. If $f(x)=c_0+c_1x+c_2x^2,$ what are the coordinates of f in this ordered basis $\mathscr{B}$? (E, CO 1)

17. Using Cramer's rule solve the following system of equations:-$x+y+z=11; 2x-6y-z=0; 3x+4y+2z=0$ (A, CO 3)

18. Let W be an invariant subspace under a linear operator T on a finite dimensional vector space V and let α be any element of V. Show that the T-conductor of α into W divides the minimal polynomial for T. (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let V be a finite- dimensional vector space over the field F. Prove that V and V^{**} are isomorphic.Further show that each basis for V^* is the dual of some basis for V. (An, CO 2)

20. (a) Let T be a linear operator on a finite-dimensional vector space V. Suppose that $T\alpha=c\alpha$. If f is any polynomial, show that $f(T)\alpha=f(c)\alpha$.
(b) Let T be a linear operator on the finite-dimensional space V.Let $c_1,\ldots,c_k$ be the distinct characteristic values of T and let W_i be the space of characteristic vectors associated with the characteristic value c_i.If $W=W_1+W_2+\ldots+W_k$, show that dim W=dim W_1+dim W_2+...+dim W_k. If $\mathscr{B}_i$ is an ordered basis for W_i, show that $\mathscr{B}$=$(\mathscr{B}_1,\mathscr{B}_2,\ldots,\mathscr{B}_k)$ is an ordered basis for W. (An)

21. Let W be the subspace of $\mathbb{C}^3$ spanned by $\alpha_1=(1,0,i)$ and $\alpha_2=(1+i,1,-1)$.
(a) Show that α_1 and α_2 form a basis for W.
(b) Show that the vectors $\beta_1=(1,1,0)$ and $\beta_2=(1,i,1+i)$ are in W and form another basis for W. (An, CO 1)
(c) What are the coordinates of α_1 and α_2 in the ordered basis $\{\beta_1,\beta_2\}$ for W?

22. Let K be a commutative ring with identity and let n be a positive integer. Prove that there exists precisely one determinant function on the set of $n\times n$ matrices over K. (A, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1		A	3, 7, 11, 16, 21	11
CO 2		An	19	5
CO 3		An	1, 5, 9, 13, 17, 22	12

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;