Reg. No .... Name **24U366**

B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS (CORE COURSE) FOR B.Sc. COMPUTER APPLICATION

COURSE : 19U3CRCMT4 : VECTOR CALCULUS, TRIGONOMETRY AND MATRICES

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Separate into real and imaginary parts the expression cot(x+iy).
2. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$
3. Determine a so that the vector $\vec{v} = (z+3y)\boldsymbol{i} + (x+2z)\,\boldsymbol{j} + (x+az)\boldsymbol{k}\ is\ solenoidal.$
4. If $\vec{r} = xi + yj + zk$, show that curl $\vec{r} = \vec{0}$
5. If **F**=3xy**i**-y^2**j**, evaluate $\int_C \mathrm{F}.\,\mathrm{d}\,\mathrm{r}$,where**C** is the arc of the parabola $y=2x^2$ from (0,0) to (1,2).
6. Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
7. Show that every square matrix is expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix.
8. Prove that $\cosh 3x = 4\cosh^3 x - 3\cosh x$.
9. Using divergence theorem evaluate $\iint_S \boldsymbol{r}.\widehat{n}dS\ where\ S\ is\ a\ closed\ surface\ and\ \vec{r}\ is\ a\ position\ vector.$
10. Find the total workdone in moving a particle in a force field given by **F**=3xy**i**-5z**j**+10x**k** along the curve $x=t^2+1, y=2t^2, z=t^3$ from t=1 to t=2
11. Find gradϕ when ϕ is given by $\phi=x^3+y^3+3xyz$.
12. Find the rank of $\begin{bmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{bmatrix}$

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the inverse of A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ by Guass Jordan method.
14. Evaluate $\iint_s F.\widehat{n}ds$,where F=18z**i**-12**j**+3y**k** and S is the surface of the plane 2x+3y+6z=12 in the first octant.
15. Evaluate the surface integral $\iint_S curlF.\widehat{n}dS$ by transforming it into a line integral ,S being that part of the surface of the paraboloid $z=1-x^2-y^2$, for which $z \geq 0$ and **F**=y**i**+z**j**+x**k.**

16. Find the unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7 \; at \; (1, \; -1, \; 2)$

17. Factorise x^6+1 into real factors.

18. If $\sin \; (\theta \; + i\phi) = \tan \; (x + iy)$, show that $\frac{\tan \; \theta}{\tan h \; \phi} = \frac{\sin \; 2x}{sinh \; 2y}$

19. Solve the system of equations by matrix method.
x + y+ z = 8, x - y + 2z = 6, 3x + 5y - 7z = 14.

20. In what direction from(3,1,-2)is the directional derivative of $\phi=x^2y^2z^4$ maximum and what is its magnitude?

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. Resolve into factors the expression x^8-2x^4cos 60°+1.

22. Verify Green's theorem in the plane for $\oint_C \; (xy \; dx + \; x^2 \; dy \,)$, where C is the curve enclosing the region bounded by the parabola y=x^2 and the line y=x.

23. $If \; \vec{A} \; and \; \vec{B} \; are \; vector \; functions \; then \; show \; that$
$$\nabla \times \left(\vec{A} \times \vec{B} \right) \; = \; \left(\nabla . \, \vec{B} \right) \vec{A} \; - \left(\nabla . \, \vec{A} \right) \vec{B} \; + \left(\vec{B} . \nabla \right) \vec{A} \; - \left(\vec{A} . \nabla \right) \vec{B}$$

24. Verify Cayley-Hamilton Theorem for the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}.

(10 x 3 = 30)