B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024
SEMESTER 3 : MATHEMATICS (COMPLEMENTARY FOR PHYSICS/CHEMISTRY)
COURSE : **19U3CPMAT3 : DIFFERENTIAL EQUATIONS, MATRICES AND TRIGONOMETRY**
(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours   Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. State the condition for consistency of a system of equations.
2. Find one of the solution of $x(y - z)p + y(z - x)q = z(x - y)$.
3. Expand $\cos^8 x$ in a series of cosine of multiples of x.
4. Separate into its real and imaginary parts the expression $\cosh(\alpha + \beta i)$.
5. Solve the differential equation $(1+x^2)dy=(1+y^2)dy$.
6. Form a partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ by eliminating the arbitrary constants.
7. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.
8. Find the integrating factor of the linear equation $(x^2+1)\frac{dy}{dx}+4xy=x$.
9. Define homogeneous equation and give the condition for trivial solution and non trivial solution.
10. Calculate the eigen values of the matrix $\begin{bmatrix} 1 & 6 \\ -3 & 7 \end{bmatrix}$.
11. Find the order of the PDE $\frac{\partial^2 u}{\partial x \partial y} - \left(\frac{\partial u}{\partial z} \right)^2 + \frac{\partial u}{\partial x} = 0$.
12. Find the integrating factor of linear equations $\frac{dy}{dx}+(\frac{2x+1}{x})y=e^{-2x}$.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the values of a and b do the system of equations
    $x + y + z = 6, x + 2y + 3z = 10, x + 2y + az = b$ have $i)$ no solution $ii)$ unique solution $iii)$more than one solution
14. Find the integral of the PDE $\frac{y^2 z}{x}p + xzq = y^2$.
15. Solve the homogeneous equation $2xy\, dy= (x^2 + 3y^2)dx$.
16. Prove that
    $$\cos^{10} \theta = \frac{1}{512}[\cos 10\theta + 10 \cos 8\theta + 45 \cos 6\theta + 120 \cos 4\theta + 210 \cos 2\theta + 126].$$
17. Solve the differential equation of the first order $xydx + \left(2x^2 + 3y^2 - 20\right)dy = 0$
18. Find the integral of the PDE $pz - qz = z^2 + (x + y)^2$.

19. Find the rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

20. Sum the series $\dfrac{c\cos\theta}{1!} + \dfrac{c^3\cos 3\theta}{3!} + \dfrac{c^5\cos 5\theta}{5!} + \cdots$ inf.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. Verify Cayley hamilton theorem for the matrix A and find A^{-1} . A= $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$.

22. Solve $(3xy^2-y^3)dx-(2xy^2-xy^2)dy=0$

23. Find the solution of $\left(z^2 - 2yz - y^2\right)p + (xy + zx)q = xy - zx.$

24. Sum the series $1 + c\cos\alpha + c^2\cos 2\alpha + c^3\cos 3\alpha + \cdots$, where c is less than unity and sum the series $c\sin\alpha + c^2\sin 2\alpha + c^3\sin 3\alpha + \cdots$, where c is less than unity.

(10 x 3 = 30)