Reg. No .... Name **24P364**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT15 : MULTIVARIATE CALCULUS**

(For Regular 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. State the inverse function theorem (U, CO 3)
2. Find the Laplace transform of $\cosh at$ (A, CO 1)
3. Show that $f_\omega = xdy + ydx$ then $\int_\gamma \omega = 0$ for every 1- surface closed curve γ. (A)
4. Give an example of an orthogonal set of functions. (R, CO 1)
5. Define a convex set. (R, CO 3)
6. Show that the components of $f'(c)$ are the dot product of the successive rows of the Jacobian matrix with the vector v. (A, CO 2)
7. Write different types of Integral Transforms (A, CO 1)
8. Explain the anticommutativity of differential forms. (U, CO 4)
9. Show that total derivative of a linear function is the function itself. (A, CO 2)
10. Define basic k-forms (U, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If S is convex and if all the partial derivatives are bounded on S, then show that $\mathbf{f}$ satisfies a Lipschitz condition on S. (A, CO 3)
12. Derive the exponential form of the Fourier Integral Theorem. (A, CO 1)
13. If f is differentiable at c, then prove that f is continuous at c. (A, CO 2)
14. Integrate $\int\limits_{I^2} e^{(x+y)}dxdy$ where $I^2 = [0,1] \times [0,2]$. (An, CO 4)
15. Find $J_f(r,\theta,z)$ where $f(r,\theta,z)$ is defined by $x = rcos\theta, y = rsin\theta, z = z$ (An, CO 4)
16. Find the gradient vector $\nabla f(x,y,z)$ at the point $(1,1,1)$ of the function $f(x,y,z) = x^2 + y^3 + z^5$. (A, CO 2)
17. Verify that the mixed partial derivatives $D_{1,2}\mathbf{f}$ and $D_{2,1}\mathbf{f}$ are equal where $\mathbf{f}(x,y) = \log(x^2+y^2), \quad (x,y) = (0,0)$. (A, CO 3)
18. Prove that $\frac{x^2}{2} = \pi x - \frac{\pi^2}{3} + 2\sum\limits_{n=1}^{\infty}\frac{\cos nx}{n^2}$ if $0 \le x \le 2\pi$. (A, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. a)Derive the matrix form of Chain rule (A, CO 2)

b)Let $f(x,y) = \begin{cases} xy\sin\dfrac{1}{x^2+y^2} & if \quad (x,y) \neq (0,0) \\ 0 & if \quad (x,y) = (0,0) \end{cases}$. Compute the gradient vector $\nabla f(x,y)$ at those points(x,y)inR^2 where it exists.

20. Suppose E is an open set in R^n, T is a C'-mapping of E into an open set $V \subset R^m$. Let ω and λ be k- and m- forms in V respectively. Then prove that
(a) $(\omega + \lambda)_T = \omega_T + \lambda_T$ if $k = m$;
(b) $(\omega \wedge \lambda)_T = \omega_T \wedge \lambda_T$;
(c) $d(\omega_T) = (d\omega)_T$ if ω is of class C' and T is of class C''. (An, CO 4)

21. Let $B = B(a; r)$ be an n-ball in R^n, let ∂B denote its boundary, $\partial B = x : ||x - a|| = r$, and let $\bar{B} = B \cup \partial B$ denote its closure. Let $f = (f_1, \ldots, f_n)$ be continuous on B, and assume that all the partial derivatives $D_j f_i(x)$ exist if $x \in B$. Assume further that $f(x) \neq f(a)$ if $x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each x in B. Then prove that $f(B)$, the image of B under f, contains an n-ball with center at $f(a)$ (A, CO 3)

22. State and prove the convolution theorem for Fourier Transforms (A, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain Weirstras theorem, otherforms of Fourierseries, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms.	A	2, 4, 7, 12, 18, 22	12
CO 2	Analyze Multivariable Differential Calculus The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of the complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule.	A	6, 9, 13, 16, 19	11
CO 3	Interpret Implicit functions and extremum problems, the mean value theorem for differentiable functions, a a sufficient condition for differentiability.	An	1, 5, 11, 17, 21	11
CO 4	Explain the Integration of Differential Forms, primitive mappings, partitions of unity, change of variables, differential forms, and Stoke's theorem.	An	8, 10, 14, 15, 20	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;