Reg. No .... Name **24P352**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT14 : ADVANCED COMPLEX ANALYSIS**

(For Regular 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Let G be an open connected subset of C. For any f in $H(G)$ there is a sequence of polynomials that converges to f in $H(G)$ then show that for any f in $H(G)$ and any closed rectifiable curve γ in G, $\int_\gamma f = 0$	(A, CO 2)
2.	State Bohr- Mollerup theorem by defining the Gamma function	(An)
3.	Find the sets K_n for which G is an open annulus	(An)
4.	Prove that an analytic function with constant imaginary part is constant	(An, CO 3)
5.	State Schwarz reflection principle	(R)
6.	If $\mathbf{w} = \boldsymbol{u(x,y) + iv(x,y)}$ is an analytic function the curves of the family $\boldsymbol{u(x,y)} = \mathbf{c_1}$ and then show that curves of the family $\boldsymbol{v(x,y)} = \mathbf{c_2}$ cut orthogonally, where $\mathbf{c_1}$ and $\mathbf{c_2}$ are varying constants.	(A, CO 3)
7.	State Hadamard factorization theorem.	(E)
8.	Find the order of $f(z) = e^{e^z}$	(A)
9.	Define genus of an entire function	(U)
10.	If for any f in $H(G)$ such that $f(z) \neq 0$ for all z in G there is a function g in$H(G)$ such that $f(z) = [g(z)]^2$. Then show that G is homeomorphic to the unit disk, where G is an open connected subset of C.	(A)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	If $\mathbf{f(z)} = \boldsymbol{u + iv}$ is a regular function of z in a domain D, then show that $\boldsymbol{\nabla^2(u^p) = p(p-1)u^{p-2}\|f'(z)\|^2}$	(An, CO 3)
12.	Show that $\mathbb{C}$ and $D = \{z/\|z\| \leq 1\}$ are homeomorphic	(A, CO 2)
13.	Show that $B(E)$ is a closed subalgebra of $C(K, \mathbb{C})$ that contains every rational function with a pole in E	(A, CO 2)
14.	Suppose $\mathscr{F} \subset C(G, \Omega)$ is equicontinuous at each point of G; then prove that $\mathscr{F}$ is equicontinuous over each compact subset of G	(A)
15.	State and prove Jensen's Formula	(U, CO 4)
16.	Let $\{a_n\}$ be a sequence in $\mathbb{C}$ such that $lim\|a_n\| = \infty$ and $a_n \neq 0$ for all $n \geq 1$. If $\{p_n\}$ is any sequence of integers such that $\sum_{n=1}^{\infty}(\frac{r}{\|a_n\|})^{p_n+1} \leq \infty$ for all $r \geq 0$ then $f(z) = \prod_{n=1}^{\infty} E_{p_n}(z/a_n)$ converges in $H(\mathbb{C})$. The function f is an entire function with zeros only at the points a_n. If z_0 occurs in the sequence $\{a_n\}$ exactly m times then prove that f has a zero at $z = z_0$ of multiplicity m. Furthermore, if $p_n = n - 1$ then $\sum_{n=1}^{\infty}(\frac{1}{\|a_n\|})^{p_n+1} \leq \infty$ will be satisfied.	(R)

17. If $\{f_n\}$ is a sequence in $H(G)$ and f belongs to $C(G,\Omega)$ such that $f_n \longrightarrow f$ then prove that f is analytic and $f_n^k \longrightarrow f^k$ for each integer $k \geq 1$. (An, CO 1)

18. Let $D = \{z : |z| < 1\}$ and suppose that $f : \bar{D} \longrightarrow C$ is a continuous function such that both Ref and Imf are harmonic. Show that $f(re^{i\theta}) = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} f(e^{it}) P_r(\theta - t)dt$ for all $re^{i\theta}$ in D. (A, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $D = \{z : |z| < 1\}$ and suppose that $f : D \longrightarrow R$ is a continuous function. Then show that there is a continuous function $u : D \longrightarrow R$ such that (U, CO 3)
$(a) u(z) = f(z)$ for z in δD
$(b) u$ is harmonic in D.

20. Prove that a set $\mathscr{F} \subset C(G,\Omega)$ is normal iff the following two conditions are satisfied: (An, CO 1)
(a) For each z in G, $\{f(z) : f \in \mathscr{F}\}$ has compact closure in Ω;
(b) $\mathscr{F}$ is equicontinuous at each point of G.

21. Let G be a region and let $\{a_j\}$ be a sequence of distinct points in G with no limit point in G; and let $\{m_j\}$ be a sequence of integers. Then prove that there is an analytic function f defined on G whose only zeros are at the points a_j ; Furthermore, a_j is a zero of multiplicity m_j. (R)

22. Let G be a region such that $G = G^*$. If $f : G_+ \cup G_o \longrightarrow \mathbb{C}$ is a continuous function which is analytic on G_+ and if $f(x)$ is real for x in G_o, then prove that there is an analytic function $g : G \longrightarrow \mathbb{C}$ such that $g(z) = f(z)$ for z in $G_+ \cup G_o$. (U, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain the space of functions, Riemann mapping theorem and Weierstrass factorization theorem.	U	17, 20	7
CO 2	Analyze Runge's Theorem, Simple connectedness, MittagLeffler's theorem, Analytic continuation and Riemann surfaces, Schwartz Reflection Principle, Analytic continuation along a path, Mondromy theorem.	An	1, 12, 13, 22	10
CO 3	Interpret Harmonic functions, Basic properties of harmonic functions and Harmonic functions on the disk.	U	4, 6, 11, 18, 19	11
CO 4	Perceive Entire functions, Jensen's formula, the genus and order of an entire function, Hadamard Factorization theorem.	U	15	2

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;